Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51295

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

First Semester

Civil Engineering

MA 1101 — MATHEMATICS — I

(Common to All Branches)

(Regulation 2008)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the eigen values of adjoint of A.

2. Find the nature of the quadratic form $x^2 + y^2 + z^2$ in four variables, without reducing into canonical form.

3. Find the equation of the sphere having the points (0, 0, 0) and (2, –2, 4) as ends of the diameter.

4. If $\frac{x}{3} = \frac{y}{4} = \frac{z}{k}$ is a generator of the cone $x^2 + y^2 - z^2 = 0$, then find the value of k.

5. What is the radius of curvature of the circle $x^2 + y^2 = 25$ at any point on it?

6. Find the envelope of the family of curves $y = mx + \frac{1}{m}$, where m is the parameter.

7. If $x^3 + y^3 = 3axy$, then find $\frac{dy}{dx}$.

8. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(r, \theta)}{\partial(x, y)}$.

9. Solve the deferential equation $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=0$.

10. Convert the given variable coefficient differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=x^2$ a constant coefficient differential equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}$, hence use it to find A^{-1}. (8)

Or

(b) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ into a canonical form by orthogonal reduction. (16)

12. (a) (i) Find the equation to the right circular cylinder with radius 5 and whose axis is $\frac{x}{2}=\frac{y}{3}=\frac{z}{6}$. (8)

(ii) Find center and radius of the circle given by $x^2+y^2+z^2+2x-2y-4z-19=0$ and $x+2y+2z+7=0$. (8)

Or

(b) (i) Find the equation to the sphere having the circle $x^2+y^2+z^2+7y-2z+2=0$; $2x+3y+4z=8$ as a great circle. (8)

(ii) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=y-4=\frac{z-5}{3}$ are coplanar. Find the plane containing them. (8)

13. (a) (i) Find the centre of curvature of the parabola $y^2=4ax$ at any point using parametric equations. (8)

(ii) Find the radius of curvature at any point of $y=\cosh\left(\frac{x}{c}\right)$. (8)

Or

(b) Find circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$. (16)

14. (a) (i) Expand $e^x \log(1+y)$ in powers of x and y up to terms of third degree. (8)

(ii) If $w = f(y-z, z-x, x-y)$, show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (8)

Or

(b) (i) Discuss the maxima and minima of the function $f(x, y) = x^3 y^2 (12 - x - y)$. (8)

(ii) If $x = u(1-v)$, $y = uv$, compute J and J' and prove $JJ' = 1$. (8)

15. (a) (i) Solve $(D^2 - 4D + 3)\, y = e^{-3x} + 2x^2$. (8)

(ii) Solve $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$ using variation of parameters. (8)

Or

(b) (i) Solve $(x^2 D^2 - xD + 1) y = \left(\frac{\log x}{x}\right)^2$. (8)

(ii) Solve $(D^2 + 4) y = \cos^2 x$. (8)
