



ST. ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY
 (Approved by AICTE New Delhi, affiliated to Anna University, Chennai)
 (An ISO 9001:2015 Certified Institution)
 ANGUCHETTYPALAYAM, PANRUTI - 607 106

Continuous Internal Assessment	CIA - III					Unit Test						
Register Number	4	2	2	1	1	7	1	0	6	0	1	6
Department	ECE						Semester		II			
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Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box

PART - A			PART - B & C										Total Marks	
Q.No.	✓	Marks	Q.No.	i		ii		iii		iii		Total Marks		
				✓	Marks		✓	Marks		✓	Marks			
					T	D		T	D		T			D
1		2	11	a								16		
2		2		b	✓	16								
3			12	a	✓	14						14		
4		1		b										
5		8	13	a								14		
6				b	✓	6		8						
7			14	a								12		
8		2		b	✓	12								
9		2	15	a								12		
10		2		b	✓	5		7						
Total		09+2	16	a								69		
				b										
Grand Total		78+2	Grand Total (in words)											
Name of the Examiner							Signature of the Examiner		P. S. S. H.					

80%

PART - B.

11. b). Given :

$$2x^2 + 5y^2 + 3z^2 + 4xy$$

$$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 10\lambda^2 + 27\lambda - 18 = 0$$

$$\lambda = 1, 3, 6.$$

To find Eigen vectors :

Eigen vectors are given by $(A - \lambda I)X = 0$

$$\begin{bmatrix} 2-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(2-\lambda)x_1 + 2x_2 + 0x_3 = 0$$

$$2x_1 + (5-\lambda)x_2 + 0x_3 = 0$$

$$0x_1 + 0x_2 + (3-\lambda)x_3 = 0$$

case i)

when $\lambda = 1$, equation

$$x_1^2 + 2y_1^2 + 0z_1^2 = 0$$

$$2x_1^2 + 4y_1^2 + 0z_1^2 = 0$$

$$0x_1^2 + 0y_1^2 + 2z_1^2 = 0$$

Solving the last two equation:

$$\frac{x^2}{4} = \frac{y^2}{-2} = \frac{z^2}{0}$$

The eigen vector corresponding to $\lambda = 1$, is

$$x_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

case ii) $\lambda = 3$

$$-x^2 + 2y^2 + 0z^2 = 0$$

$$2x^2 + 2y^2 + 0z^2 = 0$$

$$0x^2 + 0y^2 + 0z^2 = 0$$

$$\Rightarrow \frac{x^2}{0} = \frac{y^2}{0} = \frac{z^2}{-6}$$

The eigen vector corresponding to $\lambda = 3$, is

$$x_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

case iii)

when $\lambda = 6$.

$$-4x^2 + 2y^2 + 0z^2 = 0$$

$$2x^2 - y^2 + z^2 = 0$$

$$x^2 + y^2 - 3z^2 = 0$$

$$\frac{x^2}{3} = \frac{y^2}{6} = \frac{z}{0}$$

The eigenvector corresponding to $\lambda = 6$,

$$x_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

Normalized matrix

$$N = \begin{bmatrix} \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \\ 0 & 1 & 0 \end{bmatrix} ; N^T = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix}$$

$$D = N^T A N = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{bmatrix} \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Canonical form = $y^T \mathcal{D} y$

$$(y_1, y_2, y_3) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{pmatrix} x^2 \\ y^2 \\ z^2 \end{pmatrix} = 1y^2 + 3y^2 + 6y^2$$

This is the required canonical form.

12.) a) solution:

By Gauss divergence theorem.

$$\iint_S \vec{F} \cdot \hat{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv$$

Given that

$$\vec{F} = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$$

$$\nabla \cdot \vec{F} = \frac{d}{dx}(x^3) + \frac{d}{dy}(y^3) + \frac{d}{dz}(z^3) = 3x^2 + 3y^2 + 3z^2$$

$$\iiint_V \nabla \cdot \vec{F} \, dv = \int_0^a \int_0^a \int_0^a (3x^2 + 3y^2 + 3z^2) \, dx \, dy \, dz$$

$$= \int_0^a \int_0^a \left[x^3 + 3xy^2 + 3xz^2 \right]_0^a \, dy \, dz$$

$$= \int_0^a \int_0^a (a^3 + 3ay^2 + 3az^2) \, dy \, dz$$

$$= \int_0^a \left[a^3 y + ay^3 + 3ayz^2 \right]_0^a \, dz$$

$$= \int_0^a (a^4 + a^4 + 3a^2 z^2) \, dz$$

$$= \left[2a^4 z + a^2 z^3 \right]_0^a = 2a^5 + a^5 = 3a^5$$

$$\iint_S \vec{F} \cdot \vec{n} \, ds = \iint_{S_1} \vec{F} \cdot \vec{n} \, ds + \iint_{S_2} \vec{F} \cdot \vec{n} \, ds + \iint_{S_3} \vec{F} \cdot \vec{n} \, ds + \iint_{S_4} \vec{F} \cdot \vec{n} \, ds + \iint_{S_5} \vec{F} \cdot \vec{n} \, ds + \iint_{S_6} \vec{F} \cdot \vec{n} \, ds$$

$$\vec{i} \text{ \& } x = a$$

$$-\vec{i} \text{ \& } x = 0$$

$$\vec{j} \text{ \& } y = a$$

$$-\vec{j} \text{ \& } y = 0$$

$$\vec{k} \text{ \& } z = a$$

$$-\vec{k} \text{ \& } z = 0$$

Evaluation of $\iint_{S_1} \vec{F} \cdot \vec{n} \, ds$

$$\iint_{S_1} \vec{F} \cdot \vec{n} \, ds = \int_0^a \int_0^a (x^2 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot \vec{i} \, dy \, dz$$

$$= \int_0^a \int_0^a x^3 \, dy \, dz$$

(The equation of the face AEGD is $x=0$)

AEGD is $x=0$

$$= \int_0^a \int_0^a a^3 \, dy \, dz$$

$$= a^3 \int_0^a (y)_0^a \, dz$$

$$= a^4 [z]_0^a = a^5$$

$$\begin{aligned} \iint_{S_2} \vec{F} \cdot \vec{n} \, ds &= \iint_{OBFC} (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot (-\vec{i}) \, dy \, dz \\ &= - \int_0^a \int_0^a x^3 \, dy \, dz \end{aligned}$$

The equation of the face OBFC is $x=0$.

$$\therefore \iint_{S_2} \vec{F} \cdot \vec{n} \, ds = 0$$

$$\begin{aligned} \iint_{S_3} \vec{F} \cdot \vec{n} \, ds &= \iint_{EBFG} (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot \vec{j} \, dx \, dz \\ &= \int_0^a \int_0^a y^3 \, dx \, dz \end{aligned}$$

$$\begin{aligned} &= a^3 \int_0^a [x]_0^a \, dz \\ &= a^4 [z]_0^a = a^5 \end{aligned}$$

$$\begin{aligned} \iint_{S_4} \vec{F} \cdot \vec{n} \, ds &= \iint_{OADC} (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot (-\vec{j}) \, dx \, dz \\ &= - \int_0^a \int_0^a y^3 \, dx \, dz \\ &= 0 \end{aligned}$$

$$\iint_{S_5} \vec{F} \cdot \vec{n} \, ds = \iint_{DBFC} (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot \vec{k} \, dx \, dy$$

$$= a^3 \int_0^a [x]_0^a dy = a^4 [a]_0^a = a^5$$

$$\iint_{S_6} \vec{F} \cdot \vec{n} \, ds = \iint_{OAE6} (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}) \cdot (-\vec{k}) \, dx \, dy$$

$$= - \int_0^a \int_0^a z^3 \, dx \, dy$$

$$= 0$$

$$\iint_S \vec{F} \cdot \vec{n} \, ds = a^5 + 0 + a^5 + 0 + a^5 + 0 = 3a^5$$

$$\iint_S \vec{F} \cdot \vec{n} \, ds = \iiint_V \nabla \cdot \vec{F} \, dv = 3a^5$$

Hence Gauss divergence theorem is verified by ① and ②

13) b) i)

Solution: Given that

$$u = e^x x \cos y - e^x y \sin y$$

TO prove

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = e^x x \cos y + e^x \cos y - e^x y \sin y$$

$$\frac{\partial^2 u}{\partial x^2} = e^x x \cos y + e^x \cos y + e^x \cos y - e^x y \sin y$$

$$\begin{aligned}
 (i) \quad u &= e^x x \cos y + 2e^x \cos y - e^x y \sin y \\
 \frac{du}{dx} &= e^x x \sin y - e^x (1 \cdot \sin y + y \cos y) \\
 \frac{du}{dy} &= -e^x x \sin y - e^x \sin y - e^x y \cos y \\
 \frac{d^2 u}{dx^2} &= -e^x x \cos y - e^x \cos y - e^x (1 \cdot \cos y + y(-\sin y)) \\
 \frac{d^2 u}{dy^2} &= -e^x x \cos y - e^x \cos y + e^x \cos y + e^x y \sin y \\
 \frac{d^2 u}{dx^2} &= -e^x x \cos y - 2e^x \cos y + e^x y \sin y \\
 \frac{d^2 u}{dy^2} &= -e^x x \cos y - e^x \cos y + e^x \cos y + e^x y \sin y
 \end{aligned}$$

$\therefore \frac{d^2 u}{dx^2} + \frac{d^2 u}{dy^2} = 0$. Hence it is a harmonic function.

$$v = \int \left(-\frac{du}{dy} dx + \frac{du}{dx} dy \right)$$

$$v = \int \left(-\frac{du}{dy} \right) dx + \int \left(\text{Terms in } \frac{du}{dx} \right)$$

$$\begin{aligned}
 v &= \int (e^x x \sin y + e^x \sin y + e^x y \cos y) dx \\
 &= \sin y (x e^x - 1 \cdot e^x) + e^x \sin y + e^x y \cos y \\
 v &= x e^x \sin y + e^x y \cos y + c \text{ is the harmonic conjugate.}
 \end{aligned}$$

ii) Solu let $z_1 = 1$, $z_2 = i$, $z_3 = -1$,

$$\omega_1 = i \quad \omega_2 = 0 \quad \omega_3 = -i$$

The bilinear transformation is given by

$$\frac{(w_2 - w_1)(w_2 - w_3)}{(w - w_3)(w_2 - w_1)} = \frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}$$

$$\frac{(w - i)(0 + i)}{(w + i)(0 - i)} = \frac{(z - 1)(i + 1)}{(z + 1)(i - 1)}$$

$$\frac{i(w - i)}{-i(w + i)} = \frac{(z - 1)(1 + i)}{(-1)(z + 1)(1 - i)}$$

$$\frac{w - i}{w + i} = \frac{(z - 1)(1 + i)}{(z + 1)(1 - i)}$$

By componendo, dividendo,

$$\left[\frac{Nx + Dy}{Nx - Dy} = \frac{Nx + Dy}{Nx - Dy} \right]$$

$$\frac{(w - i) + (w + i)}{(w - i) - (w + i)} = \frac{(z - 1)(1 + i) + (z + 1)(1 - i)}{(z - 1)(1 + i) - (z + 1)(1 - i)}$$

$$\frac{w - i + w + i}{w - i - w - i} = \frac{z + zi - 1 - i + z - iz + 1 - i}{z + iz - 1 - i - z + iz - 1 + i}$$

$$\frac{2w}{-2i} = \frac{2z - 2i}{2iz - 2} \Rightarrow \frac{w}{-i} = \frac{z - i}{zi - 1}$$

$$w = \frac{-i(z - i)}{zi - 1} = \frac{-iz + i^2}{zi - 1} = \frac{-1 - iz}{zi - 1}$$

$w = -\frac{(1 + iz)}{(iz - 1)}$ is the required transformation

14.)

b)

Solu

Put $z = e^{i\theta}$

$$d\theta = \frac{dz}{iz}$$

$$\sin\theta = \frac{1}{2i} \left(z - \frac{1}{z} \right)$$

$$\therefore \int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta} = \int_C \frac{dz/iz}{13 + \frac{5}{2i} \left(z - \frac{1}{z} \right)}$$

C is the unit circle

$$= \int_C \frac{dz/iz}{26iz + 5z^2 - 5}$$

$$= 2 \int_C \frac{dz}{5z^2 + 26iz - 5}$$

$$\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta} = 2 \int_C \frac{dz}{5z^2 + 26iz - 5} = 2 \int_C \frac{dz}{(5z+i)(2+5i)}$$

The poles of $f(z)$ are given by

$$\left(z + \frac{i}{5} \right) (2+5i) = 0$$

$$z + \frac{i}{5} = 0 ; z + 5i = 0$$

$$z = -\frac{i}{5} , -5i$$

Among these poles, only $z = -i/5$ lies inside C.

\therefore Res

$[z = -i/5]$

$$f(z) = \lim_{z \rightarrow -i/5} [z - (-i/5)] f(z)$$

$$= \lim_{z \rightarrow -i/5} (z + i/5) \frac{2}{5z^2 + 26iz - 5}$$

$$= \lim_{z \rightarrow -i/5} (z+i/5) \frac{2}{5(z+i/5)(z+5i)}$$

$$= \frac{2}{5\left(\frac{-i}{5} + 5i\right)}$$

$$= \frac{2}{5\left(\frac{-i + 25i}{5}\right)} = \frac{1}{12i}$$

Res $f(z) = 0$ as $z = 5i$ lies outside C

By residue theorem $\int_C f(z) dz = 2\pi i \left[\frac{1}{12i} \right] = \frac{\pi}{6}$

$$\int_0^{2\pi} \frac{dx}{13+5\sin x} = \frac{\pi}{6}$$

15) b) :

i) sol

$$\mathcal{L}^{-1} \left[\frac{s}{(s^2+a^2)^2} \right] = \mathcal{L}^{-1} \left[\frac{s}{s^2+a^2} \cdot \frac{1}{s^2+a^2} \right]$$

$$= \mathcal{L}^{-1} \left[\frac{s}{s^2+a^2} \right] * \mathcal{L}^{-1} \left[\frac{1}{s^2+a^2} \right]$$

$$= \cos at * \frac{\sin at}{a}$$

$$= \frac{1}{a} \int_0^t \sin au \cos a(t-u) du$$

$$= \frac{1}{2a} \int_0^t [\sin at + \sin(2au - at)] du$$

$$\begin{aligned}
 &= \frac{1}{2a} \left[a \sin at - \frac{\cos(2at - at)}{2a} \right]_0^t \\
 &= \frac{1}{2a} \left[\left(t \sin at - \frac{\cos at}{2a} \right) - \left(0 - \frac{\cos at}{2a} \right) \right] \\
 &= \frac{t \sin at}{2a}
 \end{aligned}$$

ii)

Sol. The given differential equation.

$$y'' + 4y' + 4y = e^{-t}$$

Taking the Laplace transforms on both sides we get.

$$L(y'') + 4L(y') + 4L(y) = L(e^{-t})$$

Given that $y(0) = 0$ and $y'(0) = 0$,

taking $L(y) = \bar{y}$, we get

$$s^2 \bar{y} + 4s\bar{y} + 4\bar{y} = \frac{1}{s+1}$$

$$\therefore \bar{y} = \frac{1}{(s+1)(s^2+4s+4)}$$

$$L(y) = \frac{1}{(s+1)(s+2)^2}$$

$$\therefore y = L^{-1} = \left[\frac{1}{(s+1)(s+2)^2} \right]$$

Now by partial fractional

$$\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$1 = A(s+2)^2 + B(s+1)(s+2) + c(s+1)$$

put

$$s = -1, \quad 1 = A \Rightarrow A = 1$$

$$0 = A + B$$

$$0 = 1 + B$$

$$\therefore B = -1.$$

put

$$s = -2, \quad 1 = -c \Rightarrow c = -1$$

$$\therefore \frac{1}{(s+1)(s+2)^2} = \frac{1}{s+1} + \frac{(-1)}{s+2} + \frac{(-1)}{(s+2)^2}$$

$$\therefore y = L^{-1} \left[\frac{1}{(s+1)(s+2)^2} \right]$$

$$= L^{-1} \left[\frac{1}{s+1} \right] - L^{-1} \left[\frac{1}{s+2} \right] - L^{-1} \left[\frac{1}{(s+2)^2} \right]$$

$$= e^{-t} - e^{-2t} - e^{-2t} L^{-1} \left[\frac{1}{s^2} \right]$$

$$y(t) = e^{-t} - e^{-2t} - te^{-2t}$$

Part - A

1.

Statement: Every square matrix satisfies its own characteristic equation.

2.

$$D = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$D_1 = 1$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & & 2 \end{vmatrix} = 2 - 1 = 1$$

$$D_3 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{vmatrix} = 1(6-1) - 1(3+1) - 1(1+2) \\ = 1(5) - 1(4) - 1(3) \\ = 5 - 4 - 3 \\ = 5 - 7 \\ = -2$$

$$D_1 = D_2 = D_3 = \text{'-ve'}$$

$$(5) \quad W = \frac{2Z-5}{Z+4}$$

$$\text{Put } \boxed{W=Z}$$

$$Z = \frac{2Z-5}{Z+4} \Rightarrow Z(Z+4) = 2Z-5$$

$$Z^2 + 4Z = 2Z - 5$$

$$Z^2 + 4Z - 2Z + 5 = 0$$

$$Z^2 + 2Z + 5 = 0$$

$$(10) \quad L[t \text{ cost}]$$

$$= \frac{d}{ds} L[\text{cost}]$$

$$= \frac{d}{ds} \left(\frac{s}{s^2+1} \right) \Rightarrow \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$$

$$= \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{-s^2+1}{(s^2+1)^2}$$

$$(8) \quad f(z) = \frac{z^2}{(z-2)(z+1)^2} \quad \text{at } z = 2$$

$$\left\{ \text{Res } f(z) \right\}_{z=2} = \lim_{z \rightarrow 2} (z-2) \cdot \frac{z^2}{(z-2)(z+1)^2}$$

$$= \lim_{z \rightarrow 2} \frac{z^2}{(z+1)^2}$$

$$= \frac{(2)^2}{(2+1)^2} = \frac{4}{(3)^2} = \frac{4}{9}$$

$$(9) \quad L[\sin 5t \cdot \cos t]$$

$$\text{W.K.T } \sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\begin{aligned} \sin 5t \cos t &= \frac{1}{2} [\sin(5t+t) + \sin(5t-t)] \\ &= \frac{1}{2} [\sin(6t) + \sin(4t)] \end{aligned}$$

$$L[\sin 5t \cos t] = \frac{1}{2} L[\sin 6t + \sin 4t]$$

$$= \frac{1}{2} \left\{ L[\sin 6t] + L[\sin 4t] \right\}$$

$$= \frac{1}{2} \left\{ \frac{6}{s^2+6^2} + \frac{4}{s^2+4^2} \right\}$$

(4) unit normal

$$\hat{n} = \frac{\nabla \phi}{|\nabla \phi|}$$



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Continuous Internal Assessment	11						Unit Test					
Register Number	4	2	2	1	1	7	1	1A	4	0	2	5
Department	MECH						Semester	A				
Subject Code	MA8251			Subject Title		Engineering mathematics-I						
Date & Session	11.04.2019						No. of Pages used	19				

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				✓	Marks		✓	Marks		✓	Marks				
					T	D		T	D		T	D			
1		2	11	a											
2		2		b	✓	13							13		
3		2	12	a	✓	13							13		
4				b											
5		2	13	a											
6		1		b	✓	6		6					12		
7		1	14	a											
8				b	✓	14							14		
9			15	a	✓	6		7					13		
10				b											
Total		10	16	a											
				b											
			Total											61 + 4	
Grand Total			Grand Total (in words)		Seven one										65
Name of the Examiner				Signature of the Examiner											

75

PART-B

12. a)

By Gauss divergence theorem

$$\int \int_S \vec{F} \cdot \hat{n} ds = \int \int \int_V \nabla \cdot \vec{F} dv$$

$$\vec{F} = (x^2 - yz) \vec{i} + (y^2 - zx) \vec{j} + (z^2 - xy) \vec{k}$$

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x} (x^2 - yz) + \frac{\partial}{\partial y} (y^2 - zx) + \frac{\partial}{\partial z} (z^2 - xy)$$

$$= 2x + 2y + 2z$$

$$\int \int \int_V \nabla \cdot \vec{F} dv = \int_0^c \int_0^b \int_0^a 2(x+y+z) dx dy dz$$

$$= 2 \int_0^c \int_0^b \left[\frac{x^2}{2} + xy + xz \right]_0^a dy dz$$

$$= 2 \int_0^c \int_0^b \left(\frac{a^2}{2} + ay + az \right) dy dz$$

$$= 2 \int_0^c \left(a^2 \frac{y}{2} + a \frac{y^2}{2} + ayz \right)_0^b dz$$

$$= 2 \int_0^c \left(\frac{a^2 b}{2} + \frac{ab^2}{2} + abz \right) dz$$

$$= 2 \left[a^2 \frac{bz}{2} + \frac{ab^2}{2} z + ab \frac{z^2}{2} \right]_0^c$$

$$= 2 \left[\frac{a^2 bc}{2} + \frac{ab^2 c}{2} + \frac{abc^2}{2} \right]$$

$$= 2 \frac{abc}{2} (a+b+c)$$

$$\iiint \nabla \cdot \vec{F} \, dv = abc(a+b+c)$$

$$\begin{aligned} \iint_S \vec{F} \cdot \vec{n} \, dx = & \iint_{ABCD} \vec{F} \cdot \vec{n} \, ds + \iint_{OFGH} \vec{F} \cdot \vec{n} \, ds + \iint_{BCFE} \vec{F} \cdot \vec{n} \, ds \\ & + \iint_{OADG} \vec{F} \cdot \vec{n} \, ds + \iint_{CDGF} \vec{F} \cdot \vec{n} \, ds + \iint_{OABE} \vec{F} \cdot \vec{n} \, ds \end{aligned}$$

$$i) \iint_{ABCD} \vec{F} \cdot \vec{n} \, ds = \iint_{ABCD} ((x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}) \cdot \vec{i} \, dy \, dz$$

$$= \int_0^c \int_0^b (x^2 - yz) \, dy \, dz$$

$$= \int_0^c \int_0^b (a^2 - yz) \, dy \, dz$$

$$= \int_0^c \left(a^2 y - \frac{y^2}{2} z \right) \Big|_0^b \, dz$$

$$= \int_0^c \left(a^2 b - \frac{b^2 z}{2} \right) \, dz$$

$$= \left[a^2 b z - b^2 \frac{z^2}{4} \right]_0^c$$

$$= a^2 bc - \frac{b^2 c^2}{4}$$

$$ii) \iint_{OFGH} \vec{F} \cdot \vec{n} \, ds = \iint_{OFGH} ((x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}) \cdot (-\vec{i}) \, dy \, dz$$

$$= \int_0^c \int_0^b (yz - x^2) \, dy \, dz$$

$$= \int_0^c \int_0^b yz \, dy \, dz$$

$$= \int_0^c \left(\frac{y^2}{2} \right)_0^b z \, dz = \frac{b^2}{2} \left[\frac{z^2}{2} \right]_0^c = \frac{b^2 c^2}{4}$$

$$\text{iii) } \int_{BCFE} \int \vec{F} \cdot \vec{n} \, ds = \int_{BCFE} \int (cx^2 - yz) \vec{i} + (y^2 - zx) \vec{j} + (z^2 - xy) \vec{k} \cdot \vec{j} \, dx \, dz$$

$$= \int_0^c \int_0^a (y^2 - zx) \, dx \, dz$$

$$= \int_0^c \left(b^2 x - z \frac{x^2}{2} \right)_0^a \, dz$$

$$= \int_0^c \left(b^2 a - \frac{a^2 z}{2} \right) \, dz$$

$$= \left[b^2 a z - \frac{a^2 z^2}{4} \right]_0^c$$

$$= ab^2 c - \frac{a^2 c^2}{4}$$

$$\text{iv) } \int_{OADG} \int \vec{F} \cdot \vec{n} \, ds = \int_{OADG} \int (cx^2 - yz) \vec{i} + (y^2 - zx) \vec{j} + (z^2 - yx) \vec{k} \cdot (-\vec{j}) \, dx \, dz$$

$$= \int_{OADG} \int (zx - y^2) \, dx \, dz$$

$$= \int_0^c \int_0^a zx \, dx \, dz$$

$$= \int_0^c z \left(\frac{x^2}{2} \right)_0^a dz$$

$$= \frac{a^2}{2} \left[\frac{z^2}{2} \right]_0^c = \frac{a^2 c^2}{4}$$

$$v) \iint_{CDGF} \vec{F} \cdot \vec{n} ds = \iint_{CDGF} ((x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - yx)\vec{k}) \cdot \vec{k} dx dy$$

$$= \int_0^b \int_0^a (x^2 - xy) dx dy$$

$$= \int_0^b \int_0^a (c^2 - xy) dx dy$$

$$= \int_0^b \left(c^2 x - \frac{x^2}{2} y \right)_0^a dy$$

$$= \int_0^b \left(c^2 a - \frac{a^2}{2} y \right) dy$$

$$= \left(c^2 ay - \frac{a^2}{2} \frac{y^2}{2} \right)_0^b = abc^2 - \frac{a^2 b^2}{4}$$

$$vi) \iint_{OABE} \vec{F} \cdot \vec{n} ds = \left[\because \vec{n} = -\vec{k} \text{ and } z=0 \text{ in face OABE} \right]$$

$$= \iint_{OABE} (xy - z^2) dx dy$$

$$= \int_0^a \int_0^b xy dx dy = \frac{a^2 b^2}{4}$$

$$\begin{aligned} \iint_S \vec{F} \cdot \hat{n} ds &= a^2 bc - \frac{b^2 c^2}{4} + \frac{b^2 c^2}{4} + ab^2 c - \frac{a^2 c^2}{4} \\ &+ \frac{a^2 c^2}{4} + abc^2 - \frac{a^2 b^2}{4} + \frac{a^2 b^2}{4} \\ &= a^2 bc + ab^2 c + abc^2 \\ &= abc(a+b+c) \end{aligned}$$

$$\therefore \iint_S \vec{F} \cdot \hat{n} ds = \iiint_V \nabla \cdot \vec{F} dv$$

From (1) and (2) divergence theorem is verified.

13) b) i)

$$\text{Let } u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$$

$$\frac{\partial u}{\partial x} = \frac{(\cosh 2y - \cos 2x)(2 \cos 2x) - \sin 2x \cdot 2 \sin 2x}{(\cosh 2y - \cos 2x)^2}$$

$$\therefore \left(\frac{\partial u}{\partial x} \right)_{(z,0)} = \frac{(1 - \cos 2z) 2 \cos 2z - 2 \sin^2 2z}{(1 - \cos 2z)^2}$$

$$= \frac{2 \cos 2z - 2 \cos^2 2z - 2 \sin^2 2z}{(1 - \cos 2z)^2}$$

$$= \frac{2 \cos 2z - 2}{(1 - \cos 2z)^2}$$

$$\frac{1}{2 \sin^2 z}$$

$$\left(\frac{\partial u}{\partial x}\right)_{(z,0)} = -\operatorname{cosec}^2 z$$

$$\frac{\partial u}{\partial y} = \frac{0 - \sin 2x \cdot 2 \sinh 2y}{(\cosh 2y - \cos 2x)^2}$$

$$\left(\frac{\partial u}{\partial y}\right)_{(z,0)} = 0$$

Required analytic function is given by

$$f(z) = \int \left(\frac{\partial u}{\partial x}\right)_{(z,0)} dz - i \int \left(\frac{\partial u}{\partial y}\right)_{(z,0)} dz$$

$$= \int -\operatorname{cosec}^2 z dz$$

$$f(z) = \cot z + c$$

ii)

Given, $z_1 = 0$, $z_2 = -1$, $z_3 = i$

$w_1 = i$, $w_2 = 0$, $w_3 = \infty$

The bilinear transformation is given by,

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_1)(z_2-z_1)}$$

$$\frac{(w-i)(0-\infty)}{(w-\infty)(0-i)} = \frac{(z-0)(-1-i)}{(z-i)(-1-0)}$$

$$\frac{(w-i)}{-i} = \frac{z(-1-i)}{(z-i)(-1)} = \frac{-z(1+i)}{(-1)(z-i)}$$

$$\frac{(w-i)}{-i} = \frac{z(1+i)}{z-i}$$

$$(w-i) = \frac{-iz(1+i)}{(z-i)}$$

$$(w-i)w = \frac{-iz(1+i)}{z-i} + i$$

$$= \frac{-iz(1+i) + i(z-i)}{z-i}$$

$$= \frac{-iz + z + iz + 1}{z-i}$$

$$w = \frac{z+1}{z-i}$$

14) b)

Soln:

$$\text{Let } z = e^{i\theta}$$

$$dz = ie^{i\theta} d\theta$$

$$d\theta = \frac{dz}{iz}$$

$$\cos\theta = \frac{1}{2} \left(z + \frac{1}{z} \right)$$

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} = \int_0^{2\pi} \frac{\frac{dz}{iz}}{2 + \frac{1}{2}\left(z + \frac{1}{z}\right)} = \int_0^{2\pi} \frac{dz/iz}{2 + \frac{1}{2}\left(\frac{z^2+1}{z}\right)}$$

$$= \int_C \frac{dz}{iz \left[2 + \left(\frac{z^2+1}{2z}\right)\right]}$$

$$= \int_C \frac{dz}{iz \left[\frac{4z + z^2 + 1}{2z}\right]}$$

$$= \int_C \frac{dz}{\frac{1}{2} [4z + z^2 + 1]}$$

$$= 2/i \int_C \frac{dz}{z^2 + 4z + 1}$$

$$= 2/i \int_C f(z) dz \quad \text{--- (1)}$$

where $f(z) = \frac{1}{z^2 + 4z + 1}$

$$Df=0 \Rightarrow z^2 + 4z + 1 = 0$$

$$a=1, b=-4, c=1$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4}}{2} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(-2 \pm \sqrt{3})}{2}$$

$$\boxed{z = -2 \pm \sqrt{3}}$$

Where

$$z = -2 + \sqrt{3}$$

$$= -2 + 1.732$$

$$z = -0.268$$

$$\sqrt{3} = 1.732$$

Where

$$z = -2 - \sqrt{3}$$

$$= -2 - 1.732$$

$$z = -3.732$$

The points are, $z = -2 + \sqrt{3}$, $z = -2 - \sqrt{3}$

$z = -2 + \sqrt{3} = -0.268$ lies inside the $|z| = 1$

$z = -2 - \sqrt{3} = -3.732$ lies outside the $|z| = 1$

The residue of $f(z)$ at the point $z = -2 + \sqrt{3}$.

$$\text{Res at } (z=a) = \lim_{z \rightarrow a} (z-a) f(z)$$

$$\text{Res at } (z = -2 + \sqrt{3}) = \lim_{z \rightarrow -2 + \sqrt{3}} [z - (-2 + \sqrt{3})] \frac{1}{[z - (-2 + \sqrt{3})][z - (-2 - \sqrt{3})]}$$

$$= \lim_{z \rightarrow -2 + \sqrt{3}} \frac{1}{z - (-2 - \sqrt{3})}$$

$$= \lim_{z \rightarrow -2 + \sqrt{3}} \frac{1}{z + 2 + \sqrt{3}}$$

$$= \frac{1}{-2 + \sqrt{3} + 2 + \sqrt{3}} = \frac{1}{\sqrt{3} + \sqrt{3}}$$

$$R = \frac{1}{2\sqrt{3}}$$

$$\begin{aligned} \text{By (C.R.T)} \int_C f(z) dz &= 2\pi i R \\ &= 2\pi i \frac{1}{2\sqrt{3}} \\ &= \frac{\pi i}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \int_0^{2\pi} \frac{d\theta}{2+\cos\theta} &= \frac{2}{i} \int_C f(z) dz \quad \{\text{from } \textcircled{1}\} \\ &= \frac{2}{i} \times \frac{\pi i}{\sqrt{3}} \\ &= \frac{2\pi}{\sqrt{3}} // \end{aligned}$$

15) a) i)

soln:

$$L\left[f\left(\frac{t}{2}\right)\right] = \frac{1}{1-e^{-2as}} \int_0^P e^{-st} f(t) dt$$

This function is a periodic function in the interval $[0, 2a]$.

$$L[f(t)] = \frac{1}{1-e^{-2as}} \left\{ \int_0^a e^{-st} t dt + \int_a^{2a} e^{-st} (2a-t) dt \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ \int_0^a e^{-st} t dt + \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ \left[t \left(\frac{e^{-st}}{-s} \right) - 0 \right] \left(\frac{e^{-st}}{s^2} \right) \right]_0^a$$

$$+ \left[(2a-t) \left(\frac{e^{-st}}{-s} \right) - (-1) \left(\frac{e^{-st}}{s^2} \right) \right]_a^{2a} \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ \left[t \frac{e^{-st}}{s} - \frac{e^{-st}}{s^2} \right]_0^a + \left[-(2a-t) \frac{e^{-st}}{s} + \frac{e^{-st}}{s^2} \right]_0^a \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ \left[-a \frac{e^{-as}}{s} - \frac{e^{-as}}{s^2} \right] - \left[\frac{-1}{s^2} \right] + \left[0 + \frac{e^{-2as}}{s^2} \right] \right.$$

$$\left. - \left[-a \frac{e^{-as}}{s} + \frac{e^{-as}}{s^2} \right] \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ -a \frac{e^{-as}}{s} - \frac{e^{-as}}{s^2} + \frac{1}{s^2} + \frac{e^{-2as}}{s^2} + \frac{ae^{-as}}{s} - \frac{e^{-as}}{s^2} \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ \frac{1}{s^2} + \frac{e^{-2as}}{s^2} - \frac{2e^{-as}}{s^2} \right\}$$

$$= \frac{1}{1-e^{-2as}} \left\{ \frac{1 + e^{-2as} - 2e^{-as}}{s^2} \right\}$$

$$= \frac{[1 - e^{-as}]^2}{s^2 [1 - e^{-2as}]}$$

$$= \frac{[1 - e^{-as}]^2}{s^2 [1 + e^{-as}] [1 - e^{-as}]}$$

$$= \frac{[1 - e^{-as}]}{s^2 [1 + e^{-as}]}$$

$$= \frac{[1 - e^{-as}]}{s^2 [1 + e^{-as}]}$$

$$= \frac{[1 - e^{-as}]}{s^2 [1 + e^{-as}]}$$

$$= \frac{1}{s^2} \tanh \left(\frac{as}{2} \right)$$

ii)

$$\text{Given, } y'' - 3y' + 2y = e^{-t}, \quad y(0) = 1, \quad y'(0) = 0$$

$$y''(t) - 3y'(t) + 2y(t) = e^{-t}$$

Taking Laplace on both sides,

$$L[y''(t)] - 3L[y'(t)] + 2L[y(t)] = L[e^{-t}]$$

$$[s^2 L[y(t)] - sy(0) - y'(0)] - 3[sL[y(t)] - y(0)] + 2L[y(t)] = L[e^{-t}]$$

$$s^2 L[y(t)] - s(1) - 0 - 3[sL[y(t)] - 1] + 2L[y(t)] = \frac{1}{s+1}$$

$$s^2 L[y(t)] - s - 3sL[y(t)] + 3 + 2L[y(t)] = \frac{1}{s+1}$$

$$L[y(t)](s^2 - 3s + 2) - s + 3 = \frac{1}{s+1}$$

$$L[y(t)](s^2 - 3s + 2) = \frac{1}{s+1} + (s-3)$$

$$= \frac{1 + (s-3)(s+1)}{s+1}$$

$$= \frac{1 + s^2 + s - 3s - 3}{s+1}$$

$$= \frac{(s^2 - 2s - 2)}{s+1}$$

$$L[y(t)] = \frac{(s^2 - 2s - 2)}{(s+1)(s^2 - 3s + 2)}$$

$$L[Y(t)] = \frac{(s^2 - 2s - 2)}{(s+1)(s-1)(s-2)}$$

Now by partial fractions.

$$\frac{(s^2 - 2s - 2)}{(s+1)(s-1)(s-2)} = \frac{A}{(s+1)} + \frac{B}{(s-1)} + \frac{C}{(s-2)} \quad (S-1)$$

$$= \frac{A(s-1)(s-2) + B(s+1)(s-2) + C(s+1)(s-1)}{(s+1)(s-1)(s-2)}$$

comparing num.

$$s^2 - 2s - 2 = A(s-1)(s-2) + B(s+1)(s-2) + C(s+1)(s-1)$$

put $s = -1$, we get

$$(-1)^2 - 2(-1) - 2 = A(-1-1)(-1-2) + B(0) + C(0)$$

$$1 + 2 - 2 = A(-2)(-3)$$

$$1 = 6A$$

$$\Rightarrow 6A = 1 \quad \boxed{A = \frac{1}{6}}$$

put $s = 1$, we get

$$1 - 2 - 2 = A(0) + B(1+1)(1-2) + C(0)$$

$$-3 = B(2)(-1)$$

$$-3 = -2B \Rightarrow 3 = 2B$$

$$\Rightarrow 2B = 3$$

$$\Rightarrow \boxed{B = \frac{3}{2}}$$

put $s=2$, we get

$$2^2 - 2(2) - 2 = A(0) + B(0) + C(2+1)(2-1)$$

$$4 - 4 - 2 = C(3)(1)$$

$$-2 = 3C$$

$$\Rightarrow 3C = -2$$

$$C = -\frac{2}{3}$$

$$L[y(t)] = \frac{A}{(s+1)} + \frac{B}{(s-1)} + \frac{C}{(s-2)}$$

$$= \frac{1/6}{(s+1)} + \frac{3/2}{(s-1)} + \frac{-2/3}{(s-2)}$$

$$L[y(t)] = \frac{1}{6} \left(\frac{1}{s+1} \right) + \frac{3}{2} \left(\frac{1}{s-1} \right) - \frac{2}{3} \left(\frac{1}{s-2} \right)$$

$$y(t) = \frac{1}{6} L^{-1} \left[\frac{1}{s+1} \right] + \frac{3}{2} L^{-1} \left[\frac{1}{s-1} \right] - \frac{2}{3} L^{-1} \left[\frac{1}{s-2} \right]$$

$$y(t) = \frac{1}{6} e^{-t} + \frac{3}{2} e^t - \frac{2}{3} e^{2t}$$

ii) b)

Ans:

Given: $s_1 = 0, s_2 = 0, s_3 = 0$

$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

Step I: To find char. eqn.

$$\lambda^3 - S_1 \lambda^2 + S_2 \lambda - S_3 = 0 \rightarrow (1)$$

$$S_1 = 6 + 3 + 3$$

$$\boxed{S_1 = 12}$$

$$S_2 = \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$$

$$= (9 - 1) + (18 - 4) + (18 - 4)$$

$$= 8 + 14 + 14$$

$$\boxed{S_2 = 36}$$

$$S_3 = |A| = \text{Det } A = 32$$

$$(1) \Rightarrow \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0 \rightarrow (2)$$

This is char. eqn.

Step II: To find Eigen values:

$$\text{solve (2)} \Rightarrow \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$a = 1, b = 12, c = 36, d = -32$$

\(\therefore\) The Eigen values ;

$$\boxed{\lambda_1 = 8, \lambda_2 = 2, \lambda_3 = 2}$$

Step II) : To find Eigen-vectors!

$$\text{Solve } (A - \lambda I)X = 0$$

$$\begin{pmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(6-\lambda)x_1 - 2x_2 + 2x_3 = 0$$

$$-2x_1 + (3-\lambda)x_2 - x_3 = 0$$

$$2x_1 - x_2 + (3-\lambda)x_3 = 0$$

} \rightarrow (3)

Case i : $\lambda_1 = 8$

$$\textcircled{3} \Rightarrow -2x_1 - 2x_2 + 2x_3 = 0 \rightarrow \textcircled{4}$$

$$-2x_1 - 5x_2 - x_3 = 0 \rightarrow \textcircled{5}$$

$$2x_1 - x_2 - 5x_3 = 0 \rightarrow \textcircled{6}$$

Solve $\textcircled{4}$ & $\textcircled{5}$ we get

$$\textcircled{4} \Rightarrow \begin{matrix} -2 & -2 & 2 \\ -2 & -2 & 2 \end{matrix}$$

$$\textcircled{5} \Rightarrow \begin{matrix} -2 & -5 & -1 \\ -2 & -5 & -1 \end{matrix}$$

$$\frac{x_1}{2+10} = \frac{x_2}{-4-2} = \frac{x_3}{10-4}$$

$$\frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{6}$$

\div by 6

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

\therefore The 1st Eigen vector is $x_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

Case ii $\lambda_2 = \lambda_3 = 2$

$$3) \Rightarrow 4x_1 - 2x_2 + 2x_3 = 0 \rightarrow (7)$$

$$-2x_1 + x_2 - x_3 = 0 \rightarrow (8)$$

$$2x_1 - x_2 + x_3 = 0 \rightarrow (9)$$

$$(7) = (8) = (9) \text{ we get } 2x_1 - x_2 + x_3 = 0 \rightarrow (10)$$

I put $x_1 = 0$ in eqn (10)

$$(10) \Rightarrow 0 - x_2 + x_3 = 0 \Rightarrow -x_2 = -x_3$$

$$\frac{x_2}{1} = \frac{x_3}{1}$$

\therefore The Second Eigen vector is $x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

III Put $x_3 = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$ be the orthogonal Eigen vector with x_1 & x_2

$$x_1^T x_3 = 0$$

$$(2 \ -1 \ 1) \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0$$

$$2l - m + n = 0 \rightarrow (11)$$

$$x_2^T x_3 = 0$$

$$(0 \ 1 \ 1) \begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0$$

$$0 \cdot l + m + n = 0 \rightarrow (12)$$

Solve (11) & (12)

$$2 \quad -1 \quad 1 \quad 2 \quad -1 \quad -1$$

$$0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1$$

$$\frac{l}{-2} = \frac{m}{-2} = \frac{n}{2} \quad \div b^{-2}$$

$$\frac{l}{1} = \frac{m}{1} = \frac{n}{-1}$$

∴ The Third Eigen vector is $x_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Step 4:

verification of pairwise orthogonal.

$$x_1^T x_2 = 0 ; x_2^T x_3 = 0 ; x_3^T x_1 = 0$$

verification is completed.

Hence x_1, x_2, x_3 pairwise orthogonal

Step 5:

Form a normalized matrix N : Eigen vectors are $x_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, x_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

∴ The Normal vectors are

$$\begin{pmatrix} \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix} \quad \begin{pmatrix} 0/\sqrt{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$$

∴ Normalized matrix is

$$N = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0/\sqrt{2} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

$$N^T = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0/\sqrt{2} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

Step 6: Diagonal matrix $D = N^T A N \rightarrow$ (13)

consider,

$$AN = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} 2/\sqrt{6} & 0/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix}$$

$$= \begin{pmatrix} 16/\sqrt{6} & 0/\sqrt{2} & 2/\sqrt{3} \\ -8/\sqrt{6} & 2/\sqrt{2} & 2/\sqrt{3} \\ 8/\sqrt{6} & 2/\sqrt{2} & -2/\sqrt{3} \end{pmatrix}$$

(13) $\Rightarrow D = N^T (AN)$

$$= \begin{pmatrix} 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ 0/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 16/\sqrt{6} & 0/\sqrt{2} & 2/\sqrt{3} \\ -8/\sqrt{6} & 2/\sqrt{2} & 2/\sqrt{3} \\ 8/\sqrt{6} & 2/\sqrt{2} & -2/\sqrt{3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{32+8+8}{6} & 0 & 0 \\ 0 & \frac{0+2+2}{2} & 0 \\ 0 & 0 & \frac{2+2+2}{3} \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Step 7: To find canonical form:

$$C.F. = Y^T D Y$$

$$= (y_1 \ y_2 \ y_3) \begin{pmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= 8y_1^2 + 2y_2^2 + 2y_3^2$$

1. Sum of the Eigen values $= -1 - 1 - 1 = -3$

$$\text{Product of the Eigen values } |4| = \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= -1[1-1] - 1[-1-1] + 1[1+1]$$

$$= 2 + 2 = 4$$

2. Let $A = \begin{pmatrix} 2 & 1 & 6 \\ 1 & 3 & 0 \\ 6 & 0 & 2 \end{pmatrix}$

Have

$$D_1 = |2| = 2 \text{ +ve}$$

$$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5 \text{ +ve}$$

$$D_3 = |A| = 2(6-0) - 1(2-0) + 1(0-0) = 12 - 2 = 10 \text{ +ve}$$

$\therefore D_1, D_2, D_3$ are all +ve

\therefore the given $2-F$ is positive definite

③ $\vec{F} = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$

$$\text{div}(\text{curl}) \vec{F} = \nabla \cdot (\nabla \times \vec{F})$$

$$\text{Consider } \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= \vec{i}(0-0) - \vec{j}(0,0) + \vec{k}(0,0)$$

$$= \vec{0}$$

$$\therefore \nabla \cdot (\nabla \times \vec{F}) = \nabla \cdot \vec{0} = \vec{0}$$

5 Given $w| = f(z) = 1 + \frac{2}{z}$

Let $(z) = z$ be the fixed point

$$z = 1 + \frac{2}{z} \Rightarrow z = \frac{z+2}{z}$$

$$\Rightarrow z^2 = z + 2 \Rightarrow z^2 - z - 2 = 0$$

$$(z-2)(z+1) = 0$$

$\Rightarrow z = 2, -1$ are fixed point

6

$$u = 3x^2y + 2x^2 - y^3 - 2y^2$$

$$\therefore u_x = 6xy + 4x$$

$$u_{xx} = 6y + 4$$

$$u_y = 3x^2 - 3y^2 - 4y$$

$$u_{yy} = -6y - 4$$

$$\therefore u_{xx} + u_{yy} = 0$$

$\therefore u$ is harmonic

7) The Residue of $f(z) = \frac{z^2}{(z-2)(z+1)^2}$ at $z=2$ is

$$= \lim_{z \rightarrow 2} (z-2) \frac{z^2}{(z-2)(z+1)^2}$$

$$= \frac{2^2}{(2+1)^2} = \frac{4}{9}$$

69/100



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Continuous Internal Assessment	CIA - I				Unit Test	I, II						
Register Number	A	2	2	4	1	8	1	0	6	0	0	6
Department	ECE						Semester	II				
Subject Code	EC8451				Subject Title	EMF						
Date & Session	3.2.2020 2 FN						No. of Pages used	14				

S. Durai RAS	S. Durai RAS / 3/2/2020
Name of the Hall Superintendent	Signature of the Hall Superintendent

Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box

PART - A			PART - B & C										Total Marks	
Q.No.	✓	Marks	Q.No.	i		ii		iii		iii		Total Marks		
				✓	Marks		✓	Marks		✓	Marks			
					T	D		T	D		T			D
1	✓	2	11	a										
2	✓	2		b	✓									
3	✓	1	12	a	✓	4	✓	3						
4	✓	1		b										
5	✓	1	13	a	✓									
6	✓	2		b										
7	✓	2	14	a	✓									
8	✓	2		b										
9	✓	2	15	a										
10	✓	2		b	✓									
Total		17	16	a	✓									
				b										
											Total		51	
Grand Total	68		Grand Total (in words)											
Name of the Examiner	M. PREMA LATHA				Signature of the Examiner		M. Durai RAS							

~~68/100~~

19 = 7

Part-A

1) Charge density & its unit:

It's defined as the total charge distributed over a line (or) curve.

$$\rho_l = \lim_{\Delta l \rightarrow 0} \left(\frac{\Delta Q}{\Delta l} \right)$$

$$\rho_l = \frac{Q}{l} \text{ C/m}$$

2) Gauss Law:

Gauss's law states that the total flux out of closed surface is equal to the net charge.

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

3) Electric dipole:

An electric dipole (or) simple dipole is that two equal & opposite charges separated by a very small distance.

$$m = Qd$$

4) Ohms Law:

Considers a conductor of length L , here condition on free electrons move under the influence of an electric field E .

$$F = -eE$$

69/100



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Continuous Internal Assessment			CIA - I				Unit Test			I, II		
Register Number	A	2	2	4	1	8	1	0	6	0	0	6
Department	ECE							Semester		IV		
Subject Code	EC8451				Subject Title			EMF				
Date & Session	3.2.2020 2 FN							No. of Pages used		14		

S. Durai Raj	S. Durai Raj / 3/2/2020
Name of the Hall Superintendent	Signature of the Hall Superintendent

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PART - A			PART - B & C								Total Marks			
Q.No.	✓	Marks	Q.No.	i		ii		iii		Total Marks				
				✓	Marks		✓	Marks				✓	Marks	
					T	D		T	D				T	D
1	✓	2	11	a										
2	✓	2		b	✓							11		
3	✓	1	12	a	✓	4	✓	3				7		
4	✓	1		b										
5	✓	1	13	a	✓							6		
6	✓	2		b										
7	✓	2	14	a	✓							7		
8	✓	2		b										
9	✓	2	15	a										
10	✓	2		b	✓								8	
Total		17	16	a	✓							13		
				b										
			Total							51				
Grand Total	68		Grand Total (in words)											

Name of the Examiner	M. PREMA LATHA	Signature of the Examiner	M. Durai Raj
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68
100

10 = 7

Part-A

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4) Ohm's Law:

Consider a conductor of length L , here condition on free electrons move under the influence of an electric field E .

$$F = -eE$$

6) Gradient of Scalar fields

the Gradient of any scalar function is the maximum space function.

$$\Delta v = \frac{\partial v}{\partial x} \bar{a}_x + \frac{\partial v}{\partial y} \bar{a}_y + \frac{\partial v}{\partial z} \bar{a}_z$$

8) Poisson's equation & Laplace's equation:

point form of Gauss's law is

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla^2 v = -\frac{\rho_v}{\epsilon}$$

This is Poisson's equation.

$$\boxed{\nabla^2 v = 0}$$

part-B

11)

b) types of coordinate system:

they are,

- * Cartesian (or) rectangular co-ordinate system.
- * cylindrical co-ordinate system.
- * Spherical co-ordinate system.

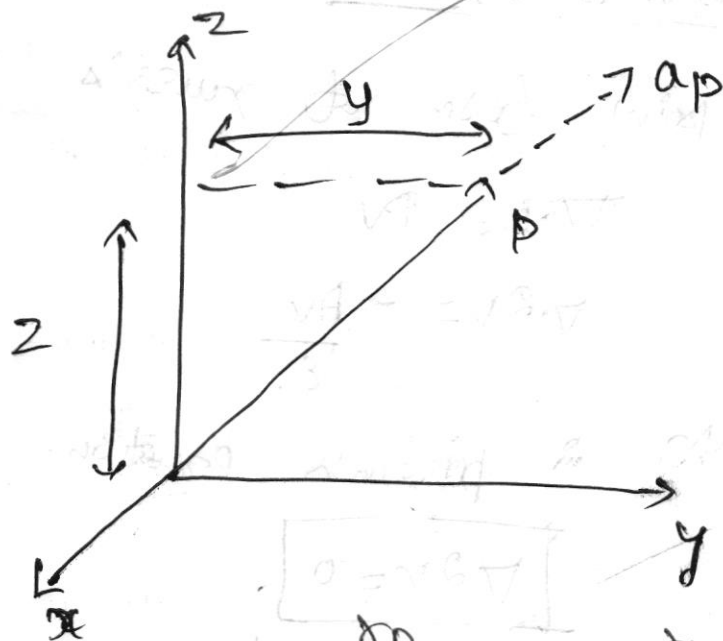
Cartesian co-ordinate System:

Unit vectors are represented as a_x , a_y & a_z .

properties:

→ position Vector.

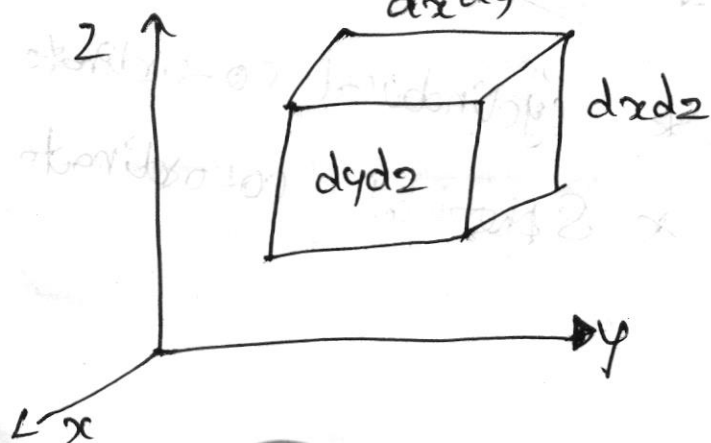
→ Each axis as $P(x, y, z)$



$$r_p = \frac{\partial p}{|\partial p|} = \frac{\partial p}{\sqrt{x^2 + y^2 + z^2}}$$

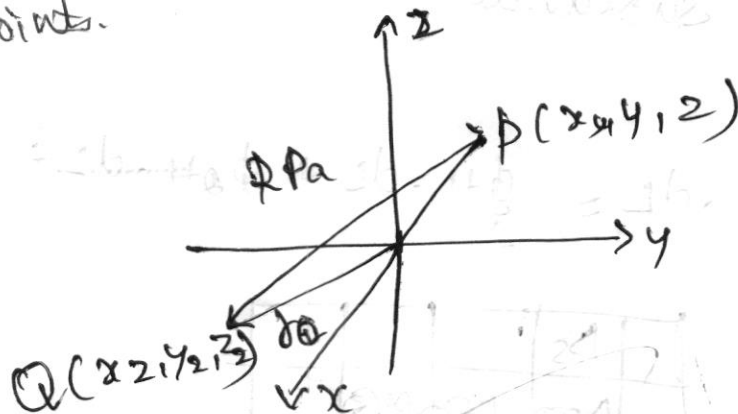
Differential Volume:

Consider a rectangular parallel piped whose length is towards x -axis.

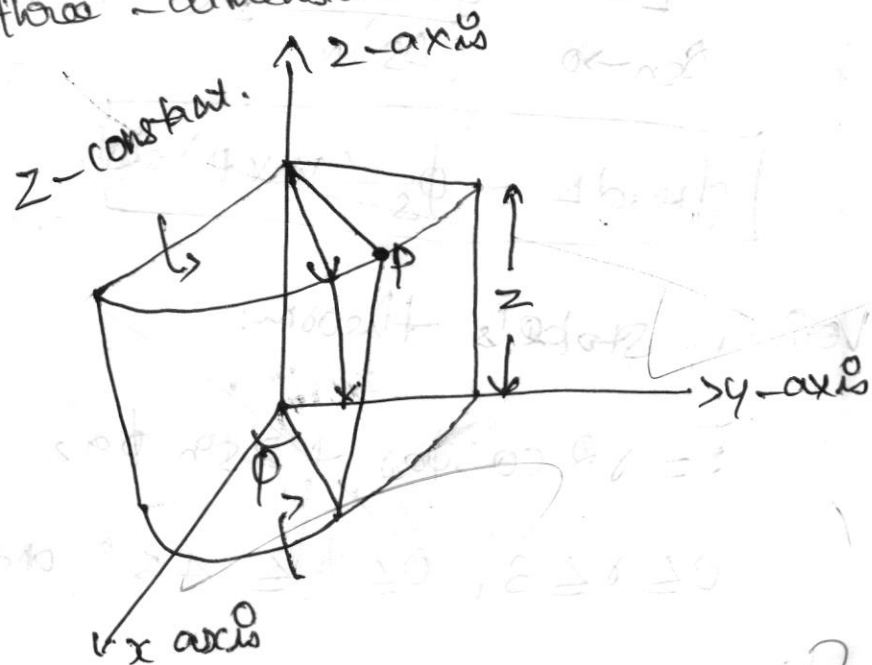


Distance Vector:

A vector formed by connecting two points.

Cylindrical coordinate System:

The cylindrical coordinate system is the three-dimensional vector.



$$\cos \phi = x/r$$

$$\boxed{x = r \cos \phi} \rightarrow \text{①}$$

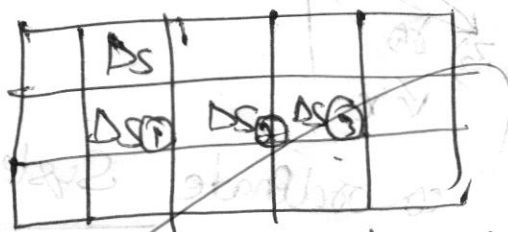
$$\sin \phi = y/r$$

$$\boxed{y = r \sin \phi} \rightarrow \text{②}$$

12) a) i) Stoke's theorem:-

Consider any surface with area 'S'
 & subdivided into different areas.

$$\oint_C \vec{H} \cdot d\vec{L} = \oint_C \vec{H} \cdot d\vec{L} + \oint_C \vec{H} \cdot d\vec{L} + \dots \rightarrow 0.$$



from the definition of curl.

$$\lim_{\Delta S \rightarrow 0} \frac{\oint_C \vec{H} \cdot d\vec{L}}{(\Delta S)_n} = (\nabla \times \vec{H})_n.$$

$$\oint_C \vec{H} \cdot d\vec{L} = \oint_S (\nabla \times \vec{H}) \cdot d\vec{S}.$$

Verify Stoke's theorem:

$$\vec{F} = r^2 \cos \phi \hat{a}_r + 2sr \sin \phi \hat{a}_z$$

$$0 \leq r \leq 3, \quad 0 \leq \phi \leq 45^\circ \text{ and } 2\pi.$$

Sol.

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{a}_r & r\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ r^2 \cos \phi & 0 & 2r \sin \phi \end{vmatrix}$$

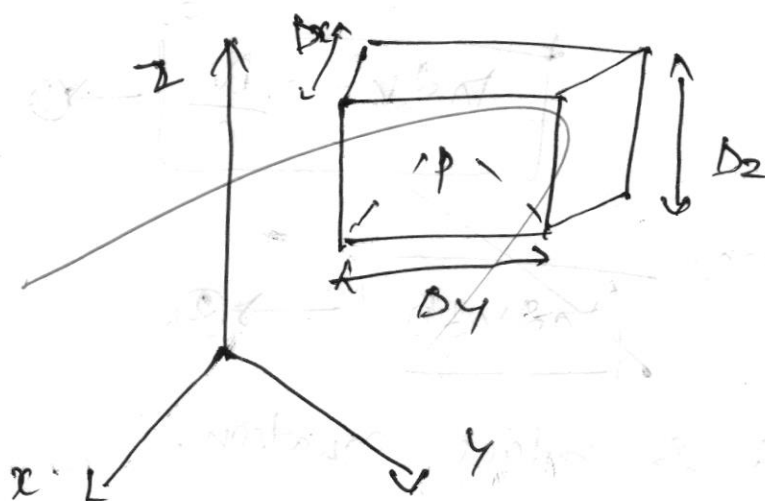
$$R.H.S = \left[\frac{r^3}{3} \right]_0^3 \left[1 - \cos \phi \right]_0^{45} = 2.636.$$

(ii) Divergence theorem:

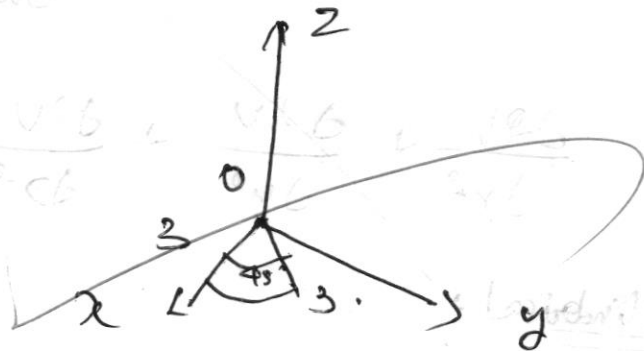
The integral of normal component of any vector over a closed surface is equal to the integral of divergence of this vector field.

proof:

Consider a point $P(x_0, y_0, z_0)$.



$$\int_{\text{vol}} \nabla \cdot \mathbf{A} \, dV = \left[\Delta x - \frac{\partial \Delta x}{\partial x} \frac{\Delta x}{2} \right] (\Delta y \Delta z)$$



$$\oint \mathbf{F} \cdot d\mathbf{r} = \iint (\nabla \times \mathbf{F}) \cdot d\mathbf{s}$$

$$\oint \mathbf{F} \cdot d\mathbf{s} = \iiint \nabla \cdot \mathbf{F} \, dV$$

14) a) Poisson's & Laplace's equations:

point form of Gauss law is

$$\nabla \cdot D = \rho_v$$

Sol. $D = \epsilon E$ & $E = -\nabla V$.

$$\nabla \cdot (\epsilon \nabla V) = \rho_v$$

$$\nabla \cdot \nabla V = -\frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \rightarrow \text{Poisson's eq.}$$

then,

$$\boxed{\nabla^2 V = 0} \rightarrow \text{Laplace's eq.}$$

this is Laplace equation.

$$\nabla \cdot D = \frac{1}{h_1 h_2 h_3} \left[\frac{d}{du} (h_2 h_3 D_u) + \frac{d}{dv} (h_3 h_1 D_v) + \frac{d}{dw} (h_1 h_2 D_w) \right]$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

for cylindrical:

$$h_1 = r, h_2 = r, h_3 = 1, u = r, v = \phi$$

or

$$\nabla^2 v = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} \left(\rho \frac{\partial v}{\partial \rho} \right) + \frac{\partial}{\partial \phi} \left(\frac{1}{\rho} \frac{\partial v}{\partial \phi} \right) + \frac{\partial}{\partial z} \left(\rho \frac{\partial v}{\partial z} \right) \right] = 0$$

$$\nabla^2 v = \frac{1}{\rho} \left(\frac{\partial}{\partial \rho} \left[\rho \cdot \frac{\partial v}{\partial \rho} \right] + \frac{1}{\rho^2} \left(\frac{\partial^2 v}{\partial \phi^2} \right) + \frac{\partial^2 v}{\partial z^2} \right) = 0$$

Spherical:

$$h_1 = r, \quad h_2 = r, \quad h_3 = r \sin \theta, \quad u > \phi.$$

$$v = 0, \quad u > \phi.$$

$$\nabla^2 v = \frac{1}{r^2 \sin \theta} \left[\frac{\partial}{\partial r} \left[r^2 \sin \theta \frac{\partial v}{\partial r} \right] + \frac{\partial}{\partial \theta} \left(-\frac{\partial \sin \theta}{\sin \theta} \frac{\partial v}{\partial \theta} \right) + \frac{\partial}{\partial \phi} \left(\frac{\partial}{\partial \sin \theta} \cdot \frac{\partial v}{\partial \phi} \right) \right]$$

$$\nabla^2 v = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2} = 0$$

$$\left(\sin \theta \frac{\partial v}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2}$$

$$+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2} = 0$$

15) a) parallel plate capacitor:

b)

It's length, this will induce charge of $-P_e$ C/m on the conductor B.

$$L = \frac{P_e}{2\pi\epsilon_0} + \frac{P_e}{2\pi\epsilon_0(d-r)} = \frac{P_e}{2\pi\epsilon_0} \left(\frac{r}{d} + \frac{r}{d-r} \right).$$

then,

$$V = - \int E \cdot dr$$

$$= - \frac{P_e}{2\pi\epsilon_0} \int_a^{\infty} \left(\frac{r}{d} + \frac{r}{d-r} \right) dr.$$

$$= - \frac{P_e}{2\pi\epsilon_0} \left(r \cdot \frac{a}{d-a} + \ln \frac{a}{d-a} \right)$$

$$= - \frac{P_e}{2\pi\epsilon_0} \left(a \ln \frac{a}{d-a} \right).$$

$$V = \frac{P_e}{\pi\epsilon_0} \ln \left(\frac{d-a}{a} \right).$$

two parallel conductors:

$$C = \frac{P_e}{V} = \frac{\pi\epsilon_0}{\ln \left(\frac{d-a}{a} \right)} \text{ F/m.}$$

$$\text{if } d \gg a, \quad C \approx \frac{\pi\epsilon_0}{\ln d/a} \text{ F/m}$$

(ii)

Capacitance of coaxial cable:

Consider a coaxial cable of inner radius 'a' and outer radius 'b', the relative dielectric filled in between two coaxial cylinders.

$$E = \frac{\rho_l}{2\pi r \epsilon}$$

then,

$$V = - \int_b^a E dr = - \frac{\rho_l}{2\pi \epsilon} \int_b^a \frac{dr}{r}$$

$$V = \frac{\rho_l}{2\pi \epsilon} \ln \left[\frac{b}{a} \right]$$

the capacitance of coaxial cable / unit length is,

$$C = \frac{\rho_l}{V} = \frac{2\pi \epsilon}{\ln(b/a)} \rho_l$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln(b/a)} \text{ f/m}$$

part-c

Electric field intensity:

Consider a charge q fixed in position & make another charge q' , call it as test charge, around.

(b)
(c)
a)

around the fixed charge, the test charge experience.

A force around the fixed charge Q , electric field is set by around the Q & any charge brought into this field.

the electric field (or) electric field intensity is defined as the electric force per unit charge.

It is given by,

$$E = F/q$$

According to Coulomb's Law

$$F = \frac{Qq}{4\pi\epsilon r^2}$$

Electric field,

$$E = F/q$$

$$E = \frac{Q}{4\pi\epsilon r^2}$$

Newton's

$$E = \frac{Q}{4\pi\epsilon r^2}$$

N/m

13) Gauss's & its applications:

a)

The electric flux passing through any closed surface is equal to the total charge enclosed by the surface.

$$\oint \vec{D} \cdot d\vec{s} = Q$$

point (or) differential form of Gauss law.
the volume charge density.

$$\nabla \cdot \vec{D} = \rho_v$$

proof of Gauss law:

Sphere gaussian surface:

$$d\phi = \vec{D} \cdot d\vec{s}$$

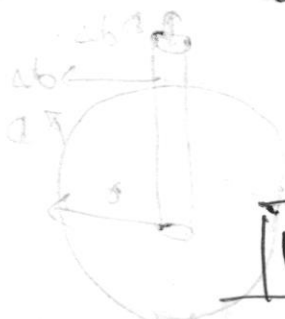
$$= D \cdot ds \cos \theta$$

$$d\phi = D ds$$

by definition,

$$D = \frac{Q}{4\pi r^2}$$

$$= 4\pi r^2 \times \frac{Q}{4\pi r^2}$$



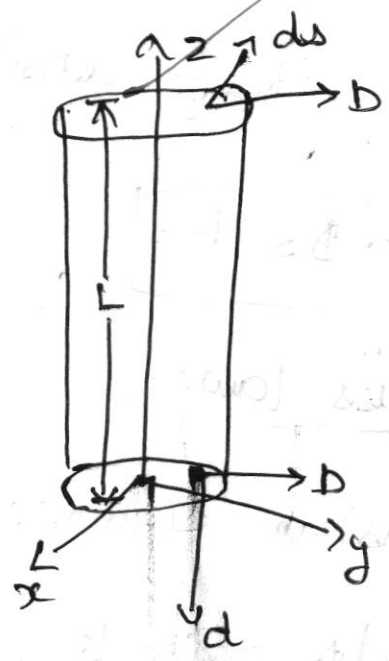
$$\oint \vec{D} \cdot d\vec{s} = Q$$

Application of Gauss law:

→ electric field intensity due to line charge.

$$D = \frac{\lambda}{2\pi r}$$

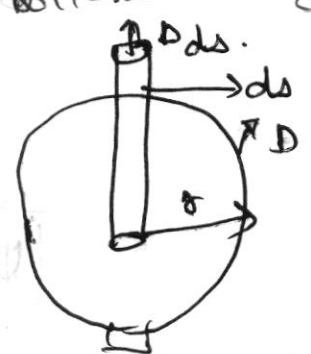
$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$



→ electric field intensity due to infinite sheet of charge.

$$\psi = \iint_{\text{top}} D \cdot ds + \iint_{\text{bottom}} D \cdot ds + \iint_{\text{sides}} D \cdot ds$$

$$d\psi = D \cdot ds$$

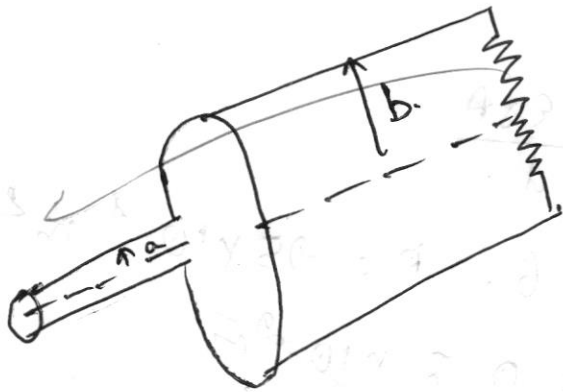


→ electric field for co-axial cable.

$$\psi = D \cdot ds \\ = D(2\pi rL)$$

$$Q = Ps(2\pi rL)$$

$$E = \frac{rL}{2\pi rL P}$$



point - A

3)

given,

$$F = 5x^2 + 8 \sin \theta.$$

$$\text{grad } \phi(x, y) = \nabla F(x, y) = \frac{\partial F}{\partial x} \hat{i} + \frac{\partial F}{\partial y} \hat{j}$$

Function $\phi(x, y)$

$$\nabla F = 5x^2 + 8 \sin \theta$$

7) Energy stored in electro magnetic field,

$$W = \left(\frac{1}{2}\right) CV^2$$

$$= \left(\frac{1}{2}\right) QV$$

$$W = \frac{Q^2}{2C}$$

9)

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Here, $\epsilon_r = 6$, $A = 25 \times 10^{-4} \text{ m}^2$

$$d = 0.5 \times 10^{-2} \text{ m}$$

$$C = 26.55 \text{ pF}$$

10)

Sol

capacitance of isolated sphere,

$$C = 4\pi\epsilon_0\epsilon_r a$$

$$= 4\pi \times 8.85 \times 10^{-12} \times 80 \times 5 \times 10^{-2}$$

$$C = 4.45 \times 10^{-10} \text{ F}$$



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Continuous Internal Assessment	CIA - II						Unit Test	-				
Register Number	A	2	2	1	1	8	1	0	4	0	0	6
Department	CSE						Semester	04				
Subject Code	CS8492		Subject Title		Database Management System							
Date & Session	29.2.2020						No. of Pages used	15				

B. Arun Kumar	B. Arun Kumar
Name of the Hall Superintendent	Signature of the Hall Superintendent

Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box													
PART - A			PART - B & C										
Q.No.	✓	Marks	Q.No.	i		ii		iii		iii		Total Marks	
				✓	Marks		✓	Marks		✓	Marks		
					T	D		T	D		T		D
1	✓	2	11	a	✓	6		✓	2+2			10+2	
2	✓	2		b									
3	✓	2	12	a									
4	✓	2		b	✓	11+1						11+1	
5	✓	2	13	a	✓	11+1						11+1	
6	✓	1+1		b									
7	✓	1+1	14	a	✓	10+2						10+2	
8	✓	1		b									
9	✓	1	15	a									
10	✓	2		b	✓	10						10	
Total	✓	16	a										
			b	✓	12+2							12+2	
			Total						64+2				
Grand Total	90	80	Grand Total (in words)		NINE ZERO								
Name of the Examiner	B. MANAVALAN				Signature of the Examiner		[Signature]						

Part - A

1. ACID properties:

- * Atomicity
- * Consistency
- * Isolation
- * Durability

2. Shadow paging:

The paging method of a file is hidden so it is called shadow paging.

3. Two-phase locking protocol:

- * This locking protocol divides the execution of a transaction into three parts.
- * The second part is where the transaction acquires all the locks.

4. * Lock based protocols

- * time stamp based protocols.

5. log-based recovery:

Log is a sequence of records, which maintains the records of actions performed by a transaction.

6. Sparse index

Index record with the largest search key.

It is slower

It is less space

dense index:

Index record appears for every search key.

It is faster.

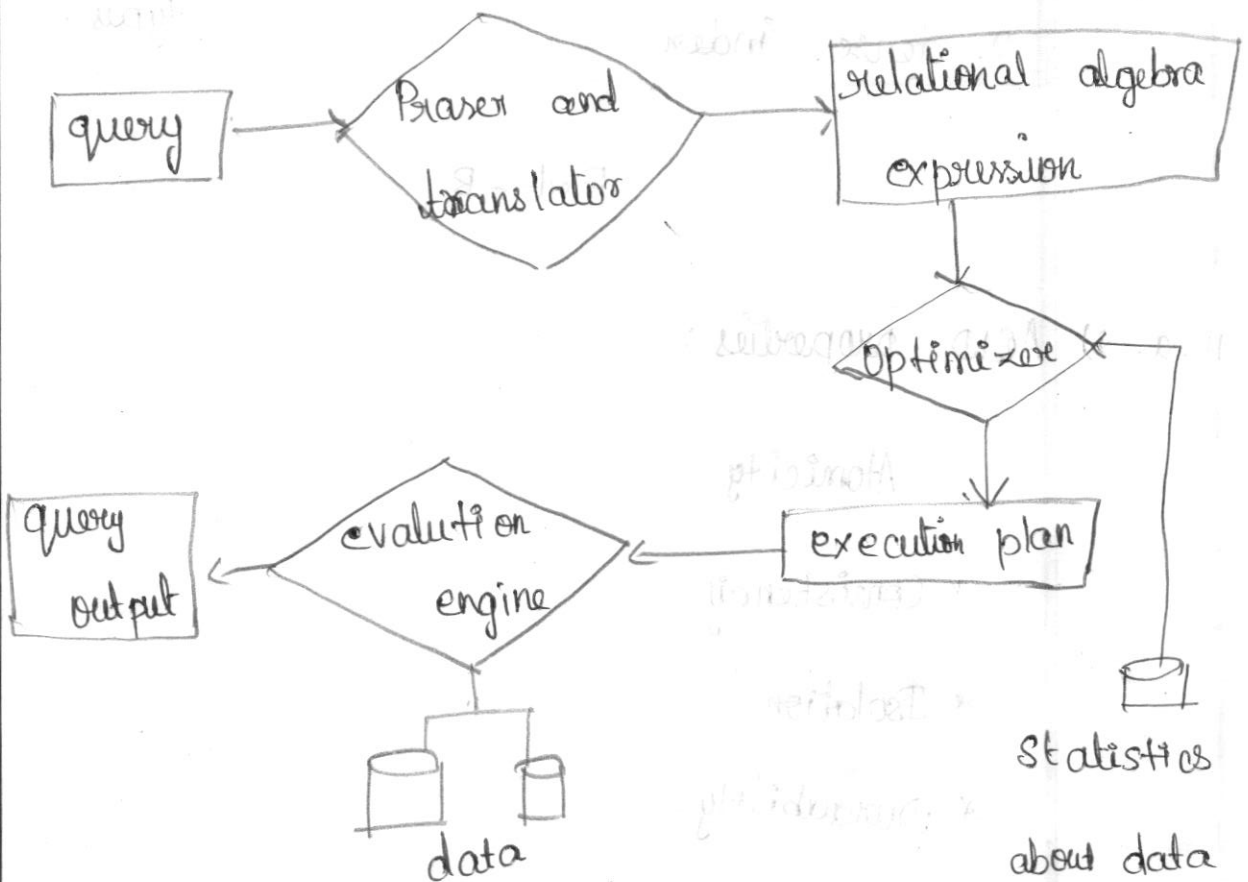
It is more space.

7. Uses of RAID:

Every write is carried out on both disks

Reads can take place from either disks.

8.



9. Flash memory:

* Temporary memory is also known as a flash memory.

* It store in temporary.

10. Primary index

Files are ordered sequentially on some search key called the primary index.

Two types

1. Sparse index
2. dense index

Secondary index

Files are unordered non-sequentially on some search key.

There are no types.

Part - B.

11. a. i) ACID properties:

- * Atomicity
- * Consistency
- * Isolation
- * Durability.

Atomicity :

A transaction must be treated as an atomic unit.

Either all of its operations are executed or none.

No partially completed transaction.

Consistency :

The database must remain in a consistent state after any transaction.

No adverse effect on the data.

Durability :

Durable enough to hold all its latest updates even if the system fails.

If a transaction commits but a system failure occurs before the data.

Isolation :

No transaction should affect the existence of any other transaction.

ii) Save point:

A SAVEPOINT is a point in a transaction when the transaction is rolled back to a certain point without rolling back the entire transaction.

Syntax:

```
SAVEPOINT SAVEPOINT_NAME;
```

Operations:

```
SQL > SAVEPOINT SP1;
```

Savepoint created

```
SQL > DELETE FROM WHERE ID=1;
```

1 row deleted.

```
SQL > SAVEPOINT SP2;
```

save point created.

Release Savepoint:

It used to remove a SAVEPOINT that is created.

Syntax:

```
RELEASE SAVEPOINT SAVEPOINT_NAME;
```

12. b)

Concurrency:

- * Multiple transaction can be run simultaneously
- * They could be interleaved.
- * Improve performance of the system.

Advantages:

Improved throughput:

- * Throughput - the average number of transaction.
- * Input/output activity can be done in parallel.

Reduced waiting time:

* Short transaction may have to wait for a concurrently active long transaction.

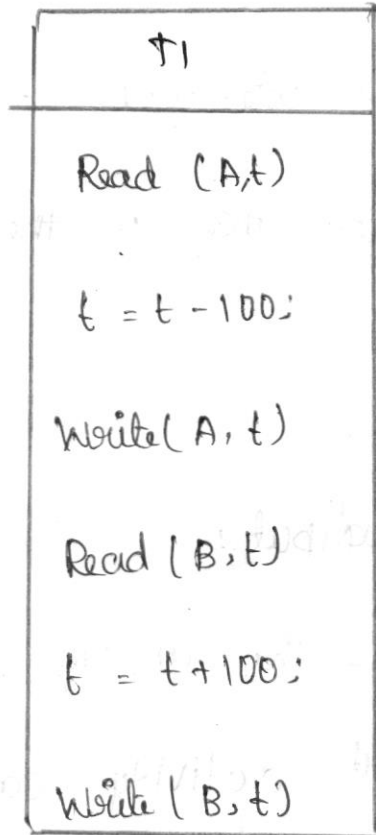
* Interleaved execution of a short transaction.

Limitation:

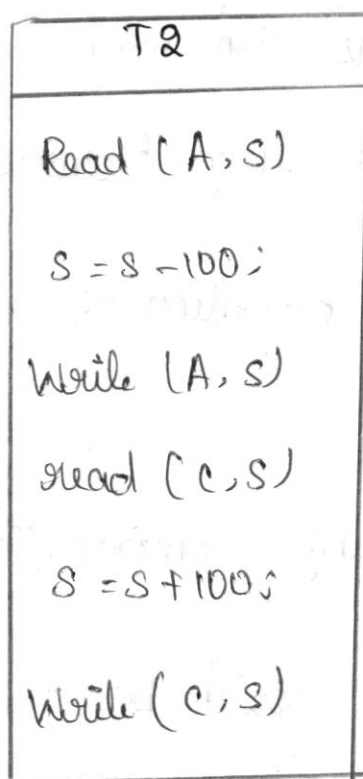
- * Creates many complication in data consistency.
- * Consistency could be compromised.

EX :

transaction 1 :



transaction 2 :



15. a) Lock Based protocols:

Data items should be accessed in mutual exclusive manner.

Database system equipped with lock based protocols.

Binary locks:

A lock on a data item can be in two states

- i) locked
- ii) unlocked.

Enforces mutual exclusion

Shared / exclusive:

Read locks are shared because on data value is being changed.

The compatibility relation between Shared (S) and Exclusive (E)

	Shared	Exclusive
Shared	True	False
Exclusive	False	False.

Two-phase locking protocols:

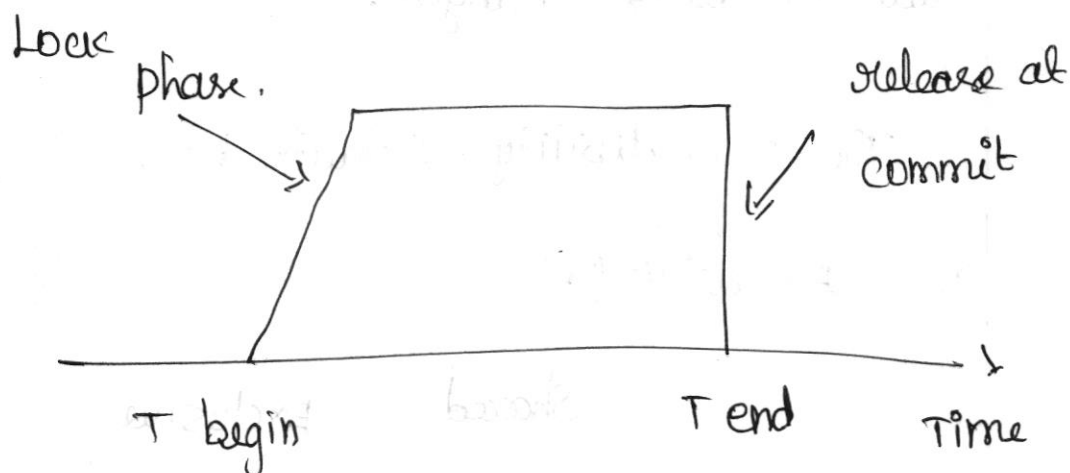
- * The protocols ensure conflict serializability
- * It does not ensure freedom from deadlocks
- * cascading rollbacks may occur.

Strict two-phase locking:

The first phase of strict-2PL is same as 2PL.

But in contrast to 2PL, strict-2PL does not release.

It does not ensure freedom from deadlocks.



14. a) RAID LEVELS:

- * RAID 0 - Non-redundant striping
- * RAID 1 - Mirrored disks.
- * RAID 2 - Memory style
- * RAID 3 - Bit parity
- * RAID 4 - Block parity
- * RAID 5 - Block Interleaved parity.
- * RAID 6 - P+Q redundancy.

RAID 0:

- * There are no parity and backup.
- * Does not employ redundancy.

~~RAI~~

RAID 1:

- * Two times the number of disks are used for storage.
- * Data is read from the disk with shorter queuing.

RAID 2:

- * Provide recovery with much lesser cost.
- * Storage efficiency increases as the number,

RAID 3:

* Subsumes level-2 - provides all the benefits of level 2 at lower cost

- * Used in application where higher bandwidth.

RAID 4:

- * This level uses block-level striping
- * provides higher I/O rates for independent block.

RAID 5:

- * Higher I/O rates compared to level 4
- * Small write requests are inefficiently compared to mirroring.

RAID 6:

- * Requires at least four disk to implement

RAID.

15. b) B+ tree :

The performance of the index-sequential file organization degrades as the file grows.

Disadvantage :

performance degrades as file grows. since many overflow blocks.

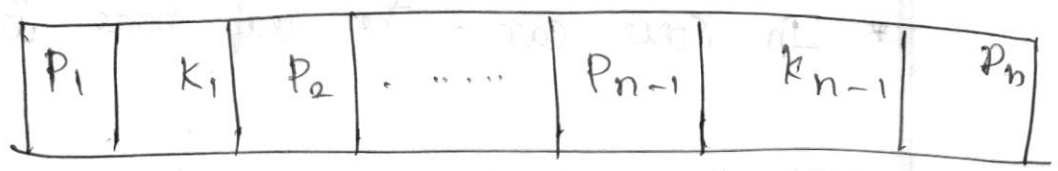
periodic reorganization of entire file is required.

Advantage :

Reorganization of entire file is not required to maintain performance.

The number of key values contained in a non-leaf node.

Leaf node is linked in order of key value.



Queries on B+ trees :

- * Follow pointer p_i to another node.
- * If $k < k_i$, follow pointer p_i . Otherwise find the appropriate pointer.
- * Eventually the pointer will point to the desired record.

Updates on B+ tree :

Insertion :

- * If the parent was already full, it will have to be split.
- * The new node must be inserted into the B+ tree

Deletion :

- * In some case, the leaf node is empty.
- * If the root becomes empty as a result,

Part - C

1b. b) Query Processing :

It is a 3-step process that transforms a high-level query.

1. parsing and translation :

Check syntax and verify relations.

Translate the query into an equivalent relationship.

2. Optimization :

Generate an optimal evaluation plan for the query plan.

3. Evaluation :

The query-execution engine takes an evaluation plan.

Distributed Query processing :

Fragmentation / replication of relations ,

Parallel execution .

* Additional communication.

Relation :

* EMP

* ASG

Query :

* High-level query

* Two possible transformation of the query.

Assume,

Data is fragmented

* Site 1 : $ASG_1 = \sigma_{EN \leq "E_3"} (ASG)$

* Site 2 : $ASG_2 = \sigma_{EN > "E_3"} (ASG)$

* Site 3 : $EM P_1 = \sigma_{EN < "E_3"} (EM P)$

* Site 4 : $EM P_2 = \sigma_{EN > "E_3"} (EM P)$

* Site 5 : result.



ST. ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY
(Approved by AICTE New Delhi, affiliated to Anna University, Chennai)
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Continuous Internal Assessment	II						Unit Test	-				
Register Number	A	2	2	1	1	8	1	0	4	0	0	6
Department	CSE						Semester	IV				
Subject Code	MA 8A02			Subject Title		Probability and Queuing theory.						
Date & Session	03-03-20 & FN						No. of Pages used	19				

S. Durairaj	S. Durairaj
Name of the Hall Superintendent	Signature of the Hall Superintendent

Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box

PART - A			PART - B & C										
Q.No.	✓	Marks	Q.No.	i		ii		iii		Total Marks			
				✓	Marks		✓	Marks		✓	Marks		
					T	D		T	D		T	D	
1	✓	2	11	a									
2	✓	2		b	✓	8		✓	8				16
3	✓	2	12	a	✓	8		✓	8				16
4	✓	2		b									
5	✓	2	13	a									
6	✓	2		b	✓	12+4							12+4
7	✓	2	14	a	✓	6+2		✓	6+2				12+4
8	✓	2		b									
9	✓	1+1	15	a	✓	5		✓	5				10
10	✓	1+1		b									
Total		18 20	16	a									
			b										
			Total							6674			
Grand Total	84 94		Grand Total (in words)	NINE		FOUR							
Name of the Examiner	Dr. Arappan			Signature of the Examiner		B							

Answer the all questions

Part - A.

1. Let us consider a random process $\{x(t)\}$ at two different time t_1 and t_2

$$\therefore E[x(t_1)] = \int_{-\infty}^{\infty} x f(x, t_1) dx.$$

$$E[x(t_2)] = \int_{-\infty}^{\infty} x f(x, t_2) dx.$$

Let $t_2 = t_1 + C$

$$\begin{aligned} \therefore E[x(t_2)] &= \int_{-\infty}^{\infty} x f(x, t_1 + C) dx \\ &= \int_{-\infty}^{\infty} x f(x, t_1) dx \\ &= E[x(t_1)] \end{aligned}$$

$$\text{Thus } E[x(t_2)] = E[x(t_1)]$$

Mean of the random process $\{x(t_1)\}$

= mean of the random processes $\{x(t_2)\}$

2. A random process $\{x(t)\}$ is called a weakly stationary process or covariance stationary process or wide-sense stationary process if.

(i) $E\{x(t)\} = \text{constant}$.

(ii) $E\{x(t)x(t+\tau)\} = R_{xx}(\tau)$ depends only on τ .

where $\tau \geq t_2 - t_1$

3. A random process $\{x(t)\}$ is called ergodic if all its ensemble averages equals appropriate time averages.

4. 2 properties of a poisson process.

1. poisson process is a Markov process.
2. let us take the conditional probability distribution of $x(t_3)$ gives the past values of $x(t_2)$ and $x(t_1)$. Assume that $t_3 > t_2 > t_1$ and $n_3 > n_2 > n_1$.

2. Additive property.
Sum of 2 independent poisson process is a poisson process.

5. The poisson process is a continuous - parameter, discrete state process which is a very useful model for many practical situations. It describes the number of times that some event has occurred, when an experiment is conducted as a function of time. In particular, these events will occur at random time.

b. We have a relation between w_s , w_q , L_s , L_q given as follows.

$$(i) L_s = \frac{\lambda}{(\mu - \lambda)} = \lambda w_s = L_q + \frac{\lambda}{\mu}$$

$$(ii) w_s = \frac{1}{(\mu - \lambda)} = w_q + \frac{1}{\mu} = \frac{L_s}{\lambda}$$

$$(iii) L_q = \frac{\lambda}{(\mu - \lambda)} = \lambda w_q$$

$$(iv) w_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

T.

1. Balking:

The reluctance of a customer to join a queue upon arrival is known as balking.

2. Reneging:

The reluctance of a customer to remain in line after joining and waiting is known as Reneging.

3. Jockeying:

If there are parallel lines, then the customers may be jockeying between lines based on the size of the queue.

8.

$$\lambda = 6 / \text{hr}$$

$$\mu = 10 / \text{hr}$$

$$t = 15 \text{ min} = \frac{1}{4} / \text{hr}$$

We have that, probability a customer has to wait for more than 15 min.

$$\therefore P(W_s > t) = e^{-(\mu - \lambda)t}$$

$$P(W_s > \frac{1}{4}) = e^{-(10-6)(\frac{1}{4})}$$

$$= e^{-1}$$

$$= 0.3679.$$

9.

Generally Queuing model may be completely specified in the following symbol form:

$$(a/b/c) : (d/e), \text{ where.}$$

a = probability law for the arrival time,

10.

b = probability law according to which the customers are being served.

c = number of service stations.

e = queue discipline.

d = the maximum number allowed in the system.

10.

$$P_n = \frac{\lambda^n}{\mu^n} P_0$$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

PART-B.

11. b) i)

To show that $\{x(t)\}$ is a wide sense stationary process we have to show that.

(i) Mean $E\{x(t)\} = \text{constant}$.

(ii) $R_{xx}(t, t+\tau)$ is a function of τ

Since θ is uniformly distributed in $[-\pi, \pi]$

$$f(\theta) = \begin{cases} \frac{1}{2\pi} & , -\pi \leq \theta \leq \pi \\ 0 & , \text{otherwise.} \end{cases}$$

$$f(\theta) = \frac{1}{2\pi} , -\pi < \theta \leq \pi \quad \rightarrow (1)$$

$$\text{Since } \{x(t)\} = A \cos(\omega t + \theta) \quad \rightarrow (2)$$

$$E\{x(t)\} = \int_{-\infty}^{\infty} x(t) f(\theta) d\theta \quad \rightarrow (3)$$

$$= \int_{-\pi}^{\pi} A \cos(\omega t + \theta) \cdot \frac{1}{2\pi} d\theta$$

$$= \frac{A}{2\pi} \int_{-\pi}^{\pi} \cos(\omega t + \theta) d\theta$$

$$= \frac{2A}{2\pi} \left[\sin(\omega t + \theta) \right]_{-\pi}^{\pi}$$

$$= \frac{A}{2\pi} \left[-\sin \omega t - \sin(-\pi - \omega t) \right]$$

$$= \frac{1}{2\pi} [-\sin \omega t + \sin \omega t]$$

$$= 0$$

\therefore a constant.

(ii) Now, $R_{xx}(t, t+\tau) = E [x(t) x(t+\tau)]$

$$E [x(t)] = \int_{-\infty}^{\infty} x(t) f(t) dt.$$

$$E [\cos(2\omega t + 2\theta + \omega\tau)] = \int_{-\pi}^{\pi} \cos(2\omega t + 2\theta + \omega\tau) \frac{1}{2\pi} dt.$$

$$= \frac{1}{2\pi} \left[\frac{\sin(2\omega t + 2\theta + \omega\tau)}{2} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi} [\sin \{2\pi + (2\omega t + \omega\tau)\} - \sin \{-2\pi + (2\omega t + \omega\tau)\}]$$

$$= \frac{1}{4\pi} [\sin(2\omega t + \omega\tau) + \sin \{2\pi - (2\omega t + \omega\tau)\}]$$

$$= \frac{1}{4\pi} [\sin(2\omega t + \omega\tau) - \sin(2\omega t + \omega\tau)].$$

$$\therefore 0.$$

$$R_{xy}(t, t+\tau) = \frac{A^2}{2} E [\cos(-\omega\tau)] + 0$$

$$= \frac{A^2}{2} E [\cos(\omega\tau)]$$

$$= \frac{A^2}{2} \cos \omega\tau, \text{ a function of } \tau$$

\therefore Hence the Auto correlation $R_{xx}(t, t+\tau)$ depends only on τ , which proves.

11. b) (i)

Formation of tpm:

The mode of transport of next day is decided on the basis of today, the travel pattern is a Markov chain, with state space,

$= \{ \text{Train}, \text{Car} \} = \{ T, C \}$, The transition probabilities of the matrix.

$$P = \begin{matrix} & \begin{matrix} T & C \end{matrix} \\ \begin{matrix} T \\ C \end{matrix} & \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \end{matrix}$$

$$P(\text{going by car}) = P(\text{getting 6 on the toss of the dice}) \\ = \frac{1}{6}$$

$$P(\text{going by train}) = 1 - \frac{1}{6} = \frac{5}{6}$$

\therefore The first day probability distribution is

$$P(1) = \left(\frac{5}{6}, \frac{1}{6} \right)$$

(i) 2nd day state probability

$$P(2) = P(1) P$$

$$= \left(\frac{5}{6}, \frac{1}{6} \right) \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \left(0 + \frac{1}{12}, \frac{5}{6} + \frac{1}{12} \right)$$

$$= \begin{pmatrix} \frac{1}{12} & \frac{4}{12} \end{pmatrix}$$

3rd day state probability distribution is

$$P(3) = P(2)P$$

$$= \begin{pmatrix} \frac{1}{12} & \frac{4}{12} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \left(0 + \frac{4}{24}, \frac{1}{12} + \frac{4}{24} \right)$$

$$= \left(\frac{11}{24}, \frac{13}{24} \right)$$

$$\therefore P = \frac{11}{24}$$

(ii) Let $\pi = (\pi_1, \pi_2)$ be the stationary state distribution of the Markov chain. By the

property of π , $\pi P = \pi$.

$$(\pi_1, \pi_2) \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = (\pi_1, \pi_2)$$

$$\pi_1 + \frac{\pi_2}{2} = \pi_2$$

$$\pi_1 + \pi_2 = 1$$

$$\pi_2 = 2\pi_1$$

$$\pi_1 + 2\pi_1 = 1 \Rightarrow 3\pi_1 = 1$$

$$\pi_1 = \frac{1}{3}, \pi_2 = 2\pi_1 = \frac{2}{3}$$

$$\therefore P = \pi_2 = \frac{2}{3}$$

12. a) y)

$$\text{Given } x(t) = \sin(\omega t + y)$$

Since 'y' is uniformly distributed r.v., we have

$$f(y) = \begin{cases} \frac{1}{2\pi} & 0 \leq y < 2\pi \\ 0 & \text{otherwise.} \end{cases}$$

Now

$$\therefore E[x(t)] = \int_{-\infty}^{\infty} x(t) f(y) dy.$$

$$= \int_0^{2\pi} \sin(\omega t + y) \cdot \frac{1}{2\pi} dy.$$

$$= \frac{1}{2\pi} [-\cos(\omega t + y)]_0^{2\pi}$$

$$= \frac{1}{2\pi} [-\cos(2\pi + \omega t) + \cos(\omega t + 0)]$$

$$= -\frac{1}{2\pi} [\cos \omega t - \cos \omega t]$$

$$\therefore E[x(t)] = 0$$

$$\text{Now } R_{xx}(\tau) = E[x(t) \cdot x(t + \tau)]$$

$$= E[\sin(\omega t + y) \times \sin\{\omega(t + \tau) + y\}]$$

$$= \frac{1}{2} E[\cos \omega t - \cos(2\omega t + \omega \tau + 2y)]$$

$$= \frac{1}{2} E(\cos \omega t) - \frac{1}{2} E(\cos(2\omega t + \omega \tau + 2y))$$

$$\text{Now } E(\cos(2\omega t + \omega \tau + 2y)) = \int_{-\infty}^{\infty} \cos(2\omega t + \omega \tau + 2y) f(y) dy$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \cos(2\omega t + \omega \tau + 2y) dy$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{\sin(2\omega t + \omega\tau + 2y)}{2} dy$$

$$= \frac{1}{4\pi} [\sin(4\pi + 2\omega t + \omega\tau) - \sin(2\omega t + \omega\tau)]$$

$$= \frac{1}{4\pi} [\sin(2\omega t + \omega\tau) - \sin(2\omega t + \omega\tau)]$$

$$= 0.$$

$$R_{xx}(\tau) = \frac{1}{2} \cos \omega\tau$$

(i) $E[x(t)] = 0 = \text{constant}$.

(ii) the auto correlation $R_{xx}(\tau)$ depends only on τ

12. a) ii)

$$\text{Let } x(t) = x_1(t) - x_2(t)$$

$$\begin{aligned} E[x(t)] &= E[x_1(t) - x_2(t)] \\ &= E[x_1(t)] - E[x_2(t)] \\ &= \lambda_1 t - \lambda_2 t \end{aligned}$$

$$E[x^2(t)] = E[(x_1(t) - x_2(t))^2]$$

$$= E[x_1^2(t) + x_2^2(t) - 2x_1(t)x_2(t)]$$

$$= E[x_1^2(t)] + E[x_2^2(t)] - 2E[x_1(t)E(x_2(t))]$$

$$= \lambda_1 t + \lambda_1^2 t^2 + \lambda_2 t + \lambda_2^2 t^2 - 2\lambda_1 t \lambda_2 t$$

$$= (\lambda_1 + \lambda_2) t + (\lambda_1 - \lambda_2)^2 t^2$$

$$E[x^2(t)] \neq (\lambda_1 - \lambda_2) t + (\lambda_1 - \lambda_2)^2 t^2$$

$$P\{X(t_3) = n_3 | X(t_2) = n_2\}$$

Hence $\{X(t)\} = \{X_1(t) - X_2(t)\}$ is not a Poisson process. Difference of 2 independent Poisson is not a Poisson process.

13. b)

- a) the average arrival rate is constant; $\lambda_n = \lambda$ for all n
- b) the average service rate is constant $\mu_n = \mu$ for all n
- c) the average arrival rate is less than the average service rate:

$\lambda < \mu$ which assures that an infinite queue will not form.

$$\text{put } \lambda_n = \lambda, \mu_n = \mu.$$

$$P_n = \left(\frac{\lambda^n}{\mu^n} \right) P_0$$

$$= \left(\frac{\lambda}{\mu} \right)^n P_0$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \left(\frac{\lambda}{\mu} \right)^n}$$

$$= \frac{1}{1 + \left(\frac{\lambda}{\mu} \right) + \left(\frac{\lambda}{\mu} \right)^2 + \dots}$$

$$= \frac{1}{\left(1 - \frac{\lambda}{\mu} \right)^{-1}}$$

$$= \left(1 - \frac{\lambda}{\mu} \right)$$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

1. Average number of customers in the system
(L_s)

$$L_s = \frac{\lambda}{\mu - \lambda} \quad \text{or} \quad \frac{\rho}{1 - \rho} \quad , \quad \rho = \frac{\lambda}{\mu}$$

2. Average number of customers in the queue
(L_q) or Average length of queue.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad \text{or} \quad \frac{\rho^2}{1 - \rho} \quad , \quad \rho = \frac{\lambda}{\mu}$$

3. Average number of customers in nonempty queue
(L_w)

$$L_w = \frac{\lambda}{\mu - \lambda}$$

4. Probability that number of customers in the system exceeds k .

$$P_{N > k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

14. a) i)

one doctor \rightarrow single channel.

Arrival of patients \rightarrow 14 (finite)

Model \rightarrow (M/M/1) : (N : FCFS)

mean arrival rate, $\lambda = 30$ per hour

Mean service rate, $\mu = 20$ per hour

capacity of the system $N = \begin{cases} \text{chairs to accommodate} \\ \text{waiting people + one chair} \end{cases}$

$$= 14 + 1 = 15$$

$$p = \frac{\lambda}{\mu} = \frac{30}{20} = \frac{3}{2}$$

$$= 1.5$$

$$a) \quad p_0 = \frac{1-p}{1-(p)^{N+1}}$$

$$= \frac{1-1.5}{1-(1.5)^{16}} = \frac{-0.5}{1-(1.5)^{16}}$$

$$\lambda' = \mu(1-p_0)$$

$$= 20 [1 - 0.00076]$$

$$= 19.98$$

20 per hour.

$$b) \quad p_0 = 0.00076$$

$$c) \quad L_S = \frac{p}{1-p} + \frac{(N+1)p^{N+1}}{1-(p)^{N+1}}$$

$$= 3 + \frac{16 \times (1.5)^{16}}{1-(1.5)^{16}}$$

$$= 13$$

$$W_S = \frac{L_S}{\text{Effective Arrival rate}}$$

$$= \frac{13}{20} = 0.65 \text{ hour}$$

$$= 39 \text{ minutes}$$

14) a) ii)

One duplicating machine \rightarrow single channel.

Arrival of jobs \rightarrow any number (∞)

Model $\rightarrow (M/M/1): (\infty / FCFS)$

Mean arrival rate, $\lambda = 5$ per hour.

Mean service time, $\frac{1}{\mu} = 6$ minutes

Mean service rate, $\mu = \frac{1}{6}$ minutes

$$= \frac{1}{6} \times 60 \text{ hour} = 10 \text{ per hour.}$$

$$(i) P = P_0 = 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{5}{10}$$

$$= \frac{1}{2}$$

\therefore percentage of idle time in the system

$$= \frac{1}{2} \times 100 = 50$$

(ii) Average time a job is waiting in the system

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{10 - 5} = \frac{1}{5} \text{ hour.}$$

(iii) Average earning per day.

$$= \text{Average no. of jobs} \times \text{earning per job.}$$

$$= 8 \times 5 \times \frac{1}{5} \times 5.$$

$$= \text{R.S. } 40/-$$

15. a) i)

Three typists \rightarrow multiple channel.

Arrived letters \rightarrow any number (∞)

Model $\rightarrow (M/M/c) : (\infty/FIFO)$

Mean arrival rate $\lambda = 15$ hour.

Mean service rate, $\mu = 6$ per hour.

No. of servers, $c = 3$

$$\frac{\lambda}{\mu} = \frac{15}{6} = \frac{5}{2}$$

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{(c\lambda/\mu)^n}{n!} + \frac{(c\lambda/\mu)^c}{c!} \times \frac{c\mu}{c\mu - \lambda}}$$

$$= \frac{1}{\sum_{n=0}^2 \frac{(5/2)^n}{n!} + \frac{(5/2)^3}{3!} \times \frac{18}{18-15}}$$

$$= \frac{1}{\frac{(5/2)^0}{0!} + \frac{(5/2)^1}{1!} + \frac{(5/2)^2}{2!} + \frac{125}{8} \times \frac{6}{3!}}$$

$$= \frac{1}{1 + \frac{5}{2} + \frac{25}{8} + \frac{125}{8}}$$

$$= 0.0449.$$

(i) The probability of all the typist will be busy.

$$P = P(N \geq 3)$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^3 P_0}{3! \left(1 - \frac{\lambda}{3\mu}\right)}$$

$$= \frac{\left(\frac{5}{2}\right)^3 \times 0.0449}{6 \left(1 - \frac{1}{3} \times \frac{5}{6}\right)}$$

$$\therefore P(N \geq 3) = 0.7066$$

2) The average number of letters waiting to be typed.

$$L_q = E(N_q)$$

$$= \frac{1}{c!c} \left(\frac{\lambda}{\mu}\right)^{c+1} \frac{1}{\left(1 - \frac{\lambda}{\mu c}\right)^2} P_0$$

$$= \frac{1}{3!3} \left(\frac{5}{2}\right)^4 \frac{1}{\left[1 - \left(\frac{5}{2}\right)\left(\frac{1}{3}\right)\right]^2} \times 0.0449$$

$$= 3.5078$$

15. a) ii)

one barber \rightarrow Single channel.

Arrival of customers \rightarrow any number (∞)

Model \rightarrow (M/M/1) : (∞ / FCFS)

Mean inter arrival time, $\frac{1}{\lambda} = 12$ minutes

Mean arrival rate, $\lambda = \frac{1}{12}$ per minutes.

Mean service time $\frac{1}{\mu} = 60$.

Mean service rate $\mu = \frac{1}{60}$ per minutes

1) Expected number of customers in the system.

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{12}}{\frac{1}{10} - \frac{1}{12}}$$

$$= \frac{\frac{1}{12}}{\frac{6-5}{60}} = \frac{1}{12} \times 60$$

$$= 5 \text{ customers.}$$

Expected number of customers in the queue.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$= \frac{1}{144}$$

$$\frac{1}{10} \left(\frac{1}{10} - \frac{1}{12} \right)$$

$$= \frac{1}{144}$$

$$\frac{1}{10} \left(\frac{6-5}{60} \right)$$

$$= \frac{1}{144} \times 600$$

$$= 4.17 \text{ customers}$$

$$\approx 4 \text{ customer.}$$

$$(ii) P = P(W > 0)$$

$$= 1 - P(W = 0)$$

$$= 1 - P(\text{Number of customers} = 0)$$

$$= 1 - P_0 = \frac{\lambda}{\mu} = \frac{\frac{1}{12}}{\frac{1}{10}} = \frac{5}{6}$$

∴ percentage of customers who have to
wait = $\frac{5}{6} \times 100 = 83.33$.



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Continuous Internal Assessment	CIA-II						Unit Test	-				
Register Number	4	2	2	1	1	8	1	0	5	0	2	1
Department	EEE						Semester					
Subject Code	EE 8401			Subject Title			Electrical machines-II					
Date & Session	3-3-2020 / FN						No. of Pages used					

<i>N. Kumar</i>	<i>N. W 3/3/2020</i>
Name of the Hall Superintendent	Signature of the Hall Superintendent

Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box

PART - A			PART - B & C										Total Marks	
Q.No.	✓	Marks	Q.No.	i		ii		iii		iii				
				✓	Marks		✓	Marks		✓	Marks			
					T	D		T	D		T	D		
1	✓	2	11	a	✓	13								13
2	✓	2		b										
3	✓	2	12	a	✓	13								13
4	✓	2		b										
5	✓	2	13	a										
6	✓	2		b	✓	2		✓	6					13
7	✓	2	14	a										
8	✓	2		b	✓	2		✓	6					13
9	✓	2	15	a										
10	✓	2		b	✓	13								13
Total	✓	20	16	a	✓	14								14
				b										
			Total										99	
Grand Total	97		Grand Total (in words)											

97

K. Srieksha

99%
V. Chand

Part-B

11a. Construction of Circle diagram.

By the given data obtained by the form of

i. No load test

ii. Blocked rotor test

iii. Stator resistance test

Step 1:

Test the reference voltage V as phasor (Y axis)

Step 2:

Select suitable current such a diameter of circle is about 20 to 30cm

Step 3:

From no load test I_0 and ϕ_0 are calculated, find ϕ_0 lagging by V as ϕ_0 . This is DO' line.

Step 4:

The horizontal of DO' is extremely to draw a line O' parallel to X -axis.

Step 5: I_{SN} is calculated by the V_0 same

side of lagging V by ϕ_{sc} . The OA is produced

Step 6:

Join O'A in the diagram

This is output line.

Step 7:

Draw the perpendicular bisector of the O'A meets the O' and extend the B. The center of the circle is C

Step 8.

Draw the Semicircle

O'C is the radius of the semicircle

C is the centre of the circle

Step 9:

Draw the perpendicular line A meets the point D midpoint of AD is E

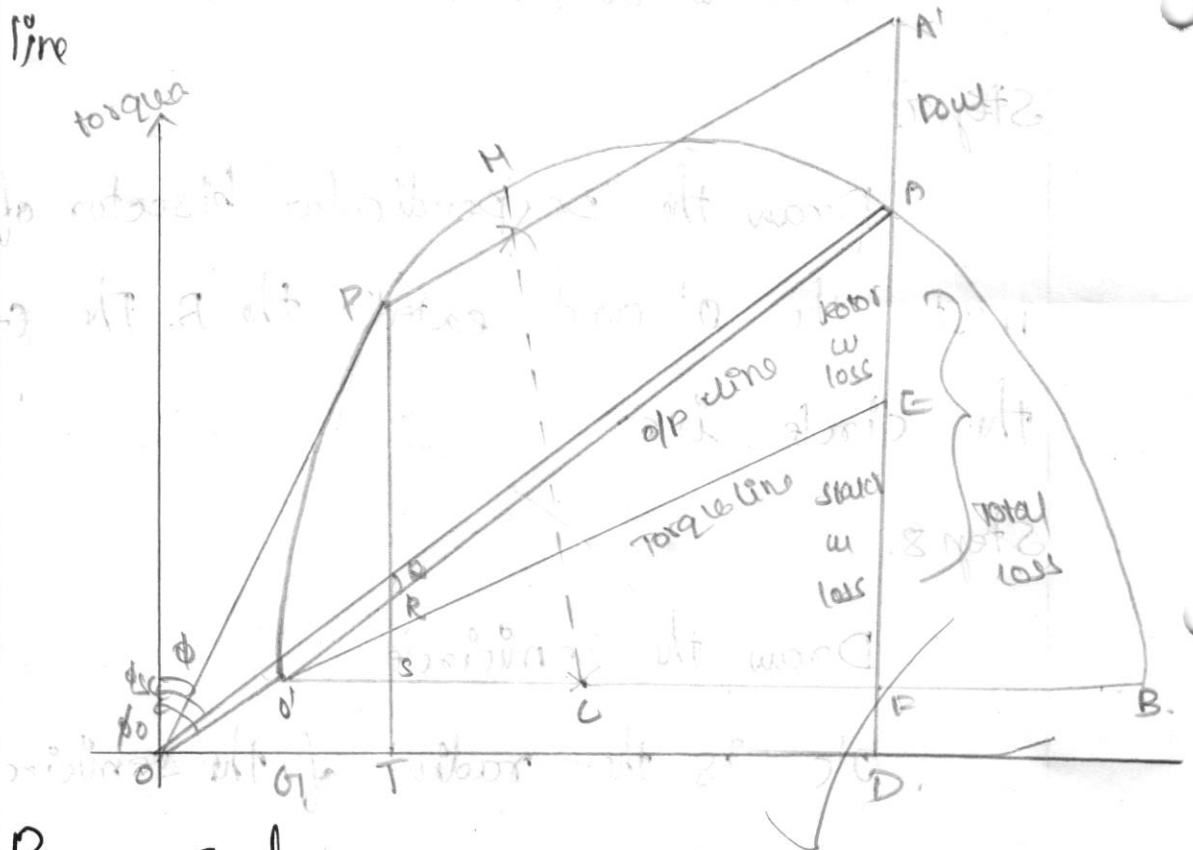
Step 10:

Torque line.

Torque is separated by the core loss and
 Sector, rotor copper loss.

$$\frac{AE}{EF} = \frac{\text{Rotor Cu loss}}{\text{Stator Cu loss}}$$

The OE line under the condition of torque



Power scale.

The power scale can be obtained in the AD'

Calculate W_{sn} of the stator output

$$\begin{aligned} \text{Power scale} &= l(AD) \times W_{sn} \text{ W/cm} \\ &= \frac{W_{sn}}{AD} \end{aligned}$$

AD is cm

Located at point E'

The stator and rotor Cu loss of the torque line is pointed in the E'

The current I_1 and I_2 is produced.

$$k = \frac{I_1}{I_2} \text{ Transformer ratio}$$

$$\frac{AE}{EF} = \frac{\text{Rotor Cu loss}}{\text{Stator Cu loss}}$$

$$= \frac{R_2 I_2^2}{R_1 I_1^2} = \frac{R_2}{R_1} \left(\frac{I_2}{I_1} \right)^2 = \frac{R_2}{R_1} \left(\frac{1}{k} \right)^2$$

Predicted performance of the circle diagram

The performance of the circle diagram is to be conducted to the power angle of point P.

The squirrel cage induction motor stator

$$\text{Cu loss} = 3 I_s N^2 \cos \phi$$

$$W_{sn} = \text{stator Cu loss} + \text{Rotor Cu loss}$$

$$\text{Rotor Cu loss} = W_{sn} - \text{Stator Cu loss}$$

$$= W_{sn} - 3 I_s N^2 \cos \phi$$

Predicted performance:-

$$\text{Power factor} = \frac{PT}{OP}$$

$$\text{Stator Cu loss} = SR \times \text{power scale}$$

$$\text{Rotor Cu loss} = QR \times \text{power scale}$$

$$\text{Total loss} = PT \times \text{power scale}$$

$$\text{Motor efficiency} = \frac{PA}{PT}$$

$$\text{Rotor efficiency} = \frac{PA}{QR}$$

Maximum quantities

* Maximum input

* Maximum output

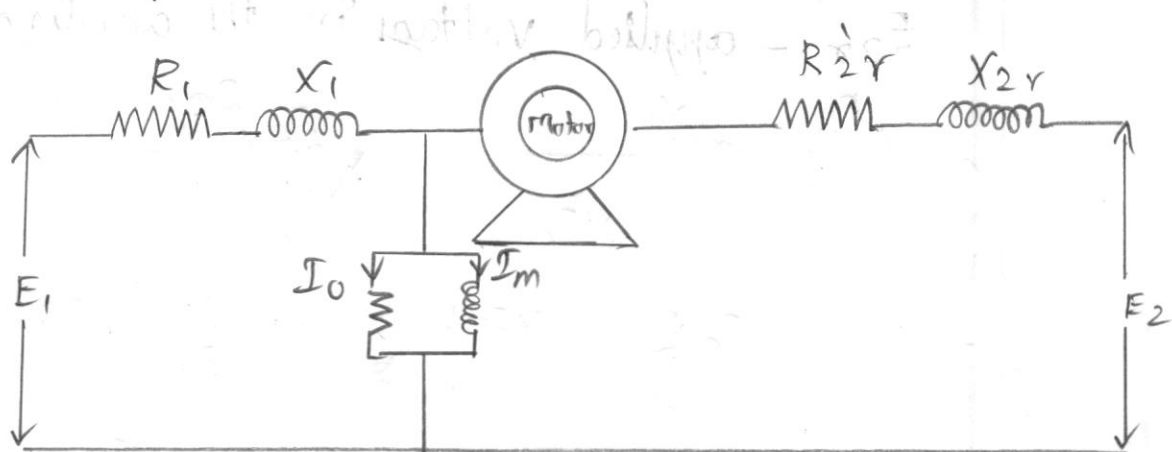
* Maximum power factor

* Maximum torque

* Starting torque

12a Equivalent Circuit

Three phase induction motor



* The equivalent circuit of three phase induction motor

* The supply to the stator to motor winding

* The circuit transfer to the stator to rotor winding there is equivalent circuit is the.

E_1 - Induced voltage is stator side

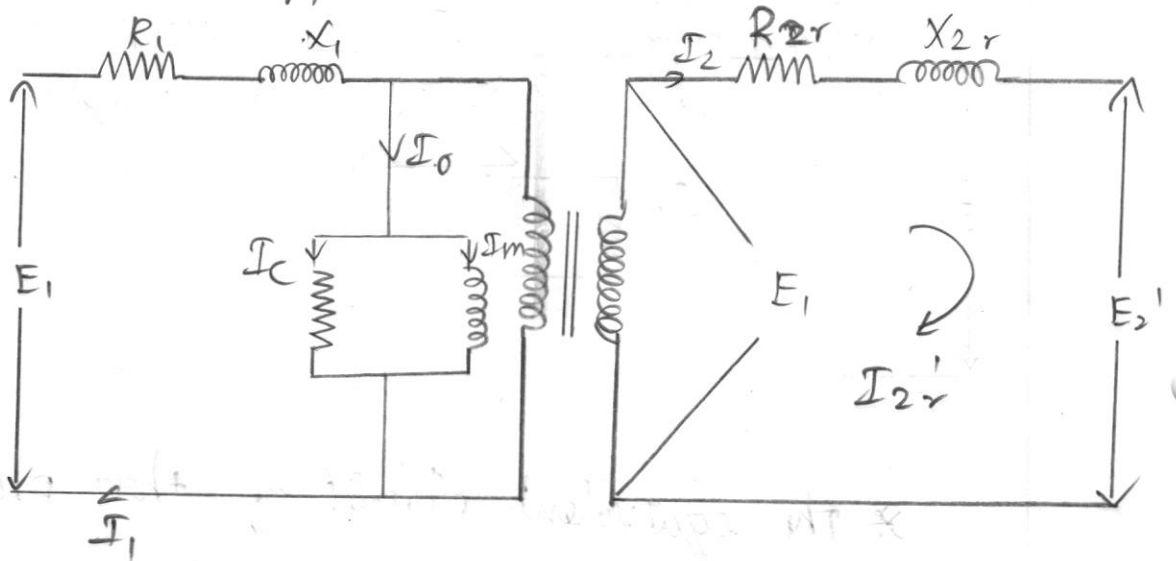
E_2 - Induced emf in the rotor

* The rotor side of equivalent circuit is the

$$= \frac{\text{rotor turns}}{\text{Stator turns}} = \frac{E_1}{E_2}$$

* The induced emf in the stator winding

E_{2r} - applied voltage in the winding.



$$I_{2r}' = \frac{E_2}{E_1}$$

$$I_{2r}' = \frac{E_{2r}}{Z_{2r}}$$

$$= \frac{SE_2^2}{\sqrt{R_2^2 + (SX_2^2)}}$$

$$I_{2r}' = \frac{E_2^2}{\sqrt{\left(\frac{R_2}{S}\right)^2 + X_2^2}}$$

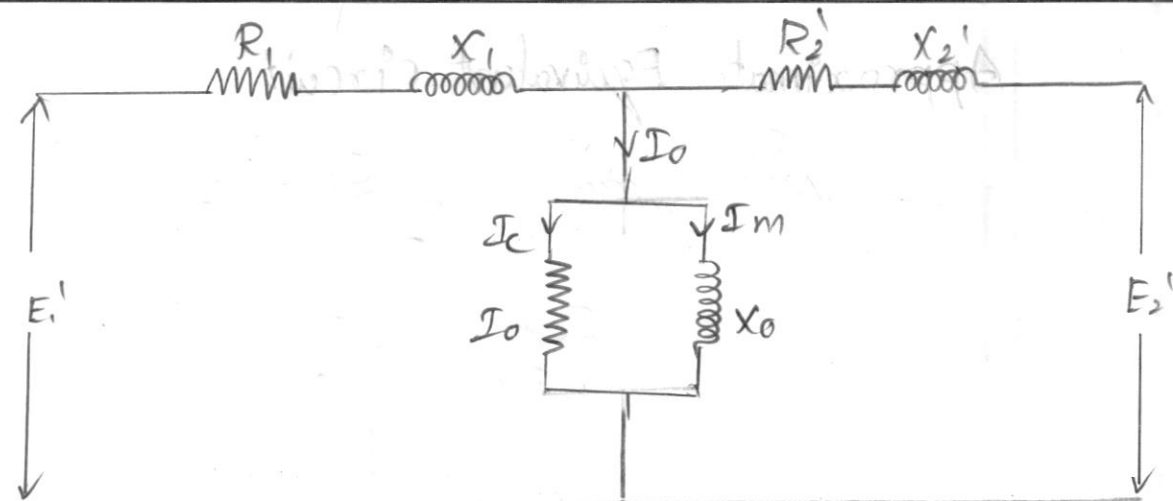
I_{2r}' = Current through the stator side

R_1 - stator resistance

X_1 - stator reactance

R_2 - Rotor resistance

X_2 - rotor reactance



$$I_o = \text{constant loss current} = \frac{V_1}{I_c}$$

$$X_o = \text{Magnetising current} = \frac{V_1}{I_m}$$

the rotor current referred to stator $I_2' = k I_1$

the rotor resistance referred to $R_2' = \frac{R_1}{k^2}$

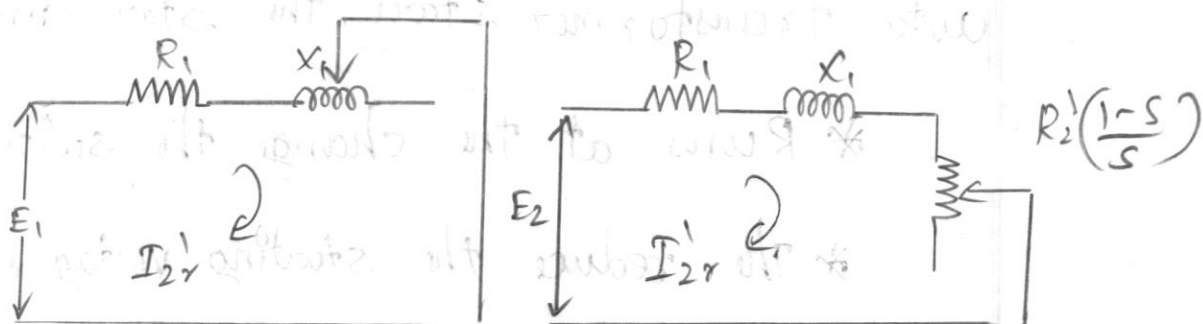
the rotor reactance referred to $X_2' = \frac{X_1}{k^2}$

The rotor resistance load referred to $R_2 = \frac{R_2'}{k}$

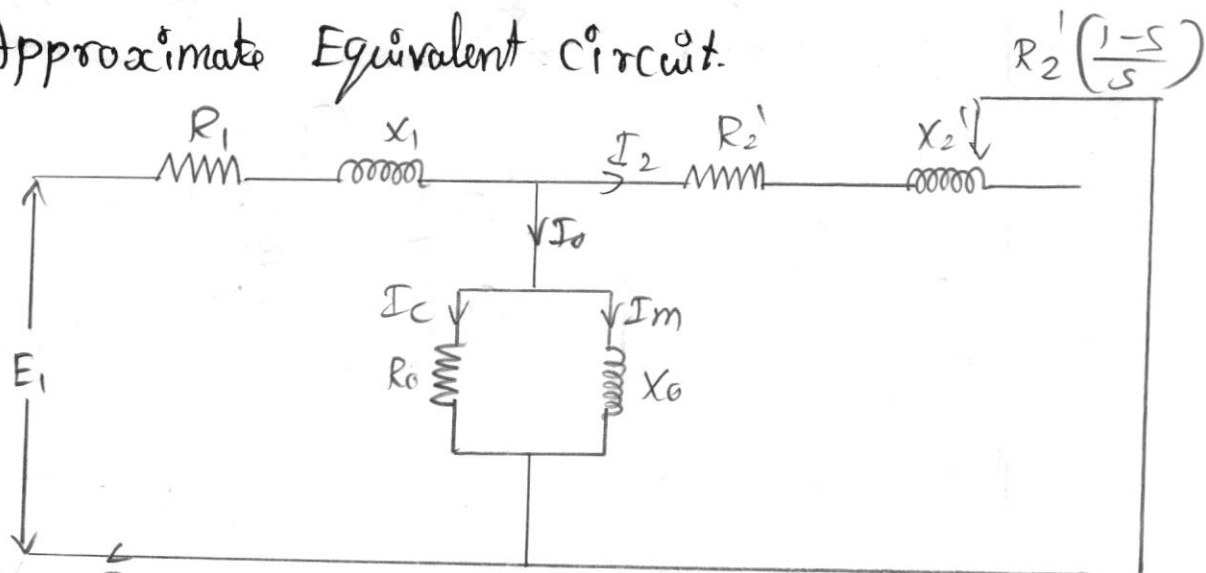
$$= \frac{R}{k^2}$$

$$= R_1 \left(\frac{1-s}{s} \right)$$

$$= R_2 + R_2 \left(\frac{1-s}{s} \right)$$



Approximate Equivalent Circuit.



This is the approximate equivalent circuit of the three phase induction motor.

136. i. Auto Transformer Starter.

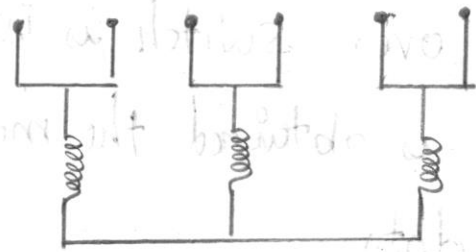
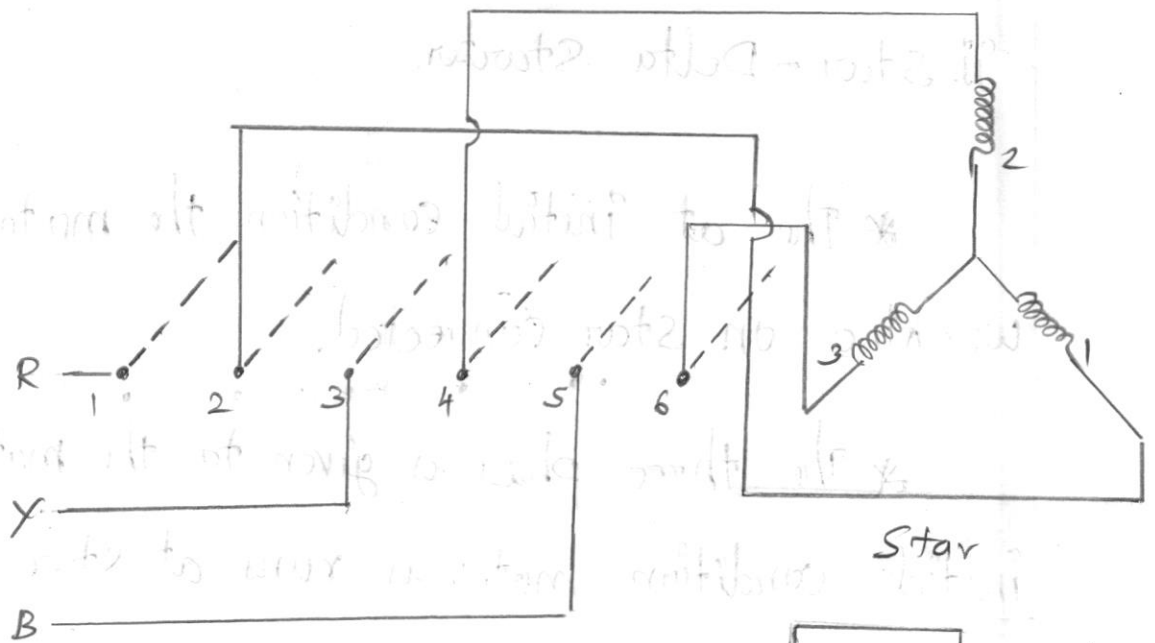
* Auto transformer is used for the reduce the voltage that is called auto transformer Starter.

* It is contained change over switch on the circuit.

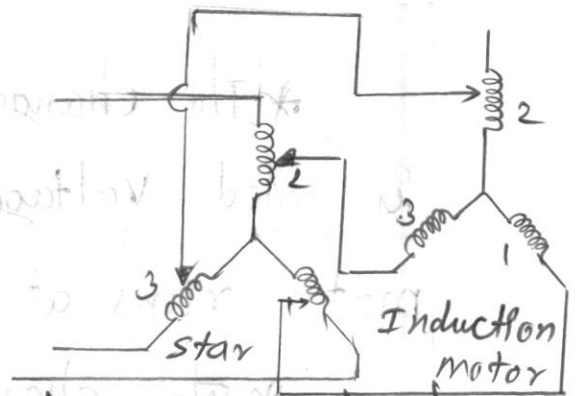
* It is given to the 3 phase supply to the auto transformer start the star connection.

* Runs at the change the switch

* The reduce the starting voltage of speed of the turns motor. then the 20% of the speed.



Auto transformer



* Motor at the change switch thrown the turn on the supply at speed.

* The speed of the auto transformer is the there is used for the auto transformer start.

* It is expensive compare the rotor resistance

ii. Star-Delta Starter.

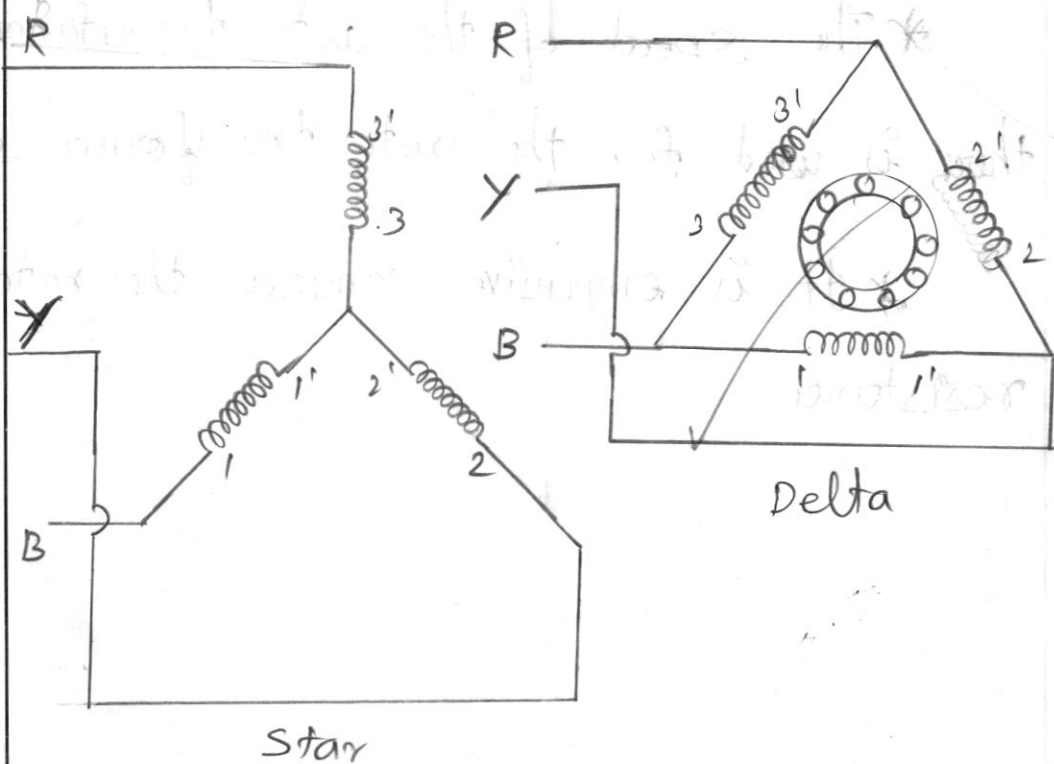
* The at initial condition the motor is working on star connected.

* The three phase is given to the motor at initial condition motor is runs at star connected.

* The change over switch is interval connected is rated voltage is obtained the motor then the motor runs at delta

* The change switch turn on through circuit.

* Then motor will turn on the delta connection.



The Star-Delta Connection motor is to

* The at normal speed at the motor on the start.

* The star - Delta starter is the working at load of the condition

* The star to change over switch is thrown on the condition of the starter working.

14b. i. Static Kramer Variable speed

* The static Kramer variable speed is the resistance of the constant to the speed.

* The applied voltage increase the resistance the speed is controlled.

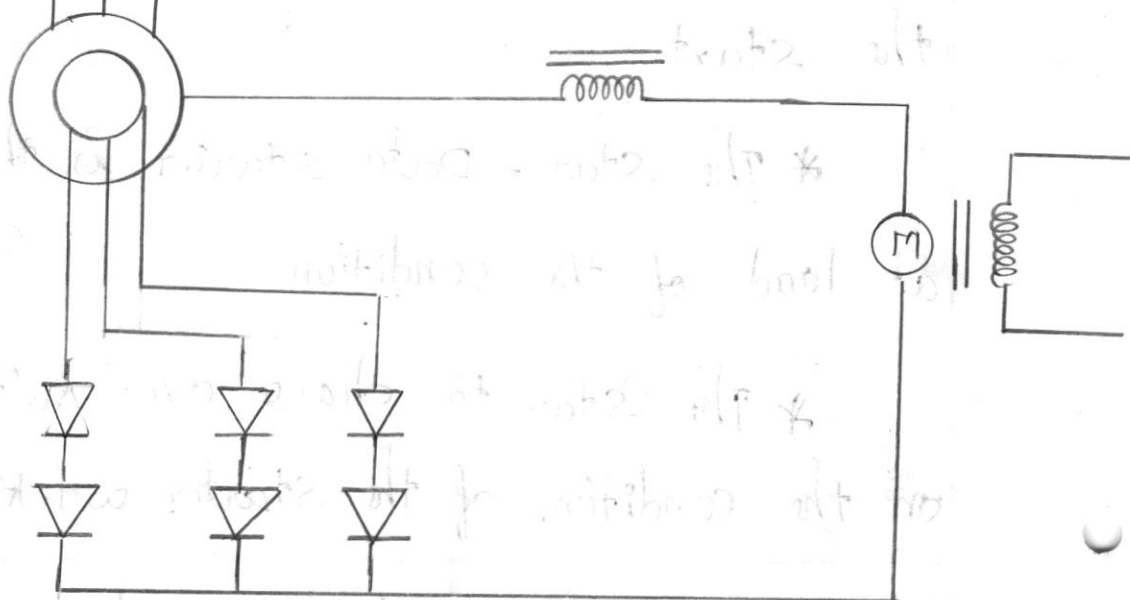
* Then the kramer static device is used in the working of the speed in controlled.

* The diode are used in this type of working

* The voltage is applied on the device resistance the varied in the speed is controlled.

* This system is advanced.

3 ϕ Supply



* The Kramer variable speed of the motor is the should resistance is add.

* The speed is proportional to the No of poles

* In this method is used for the speed control method

* This method can be special type of method in this type speed can controlled.

ii. DC Dynamic Braking

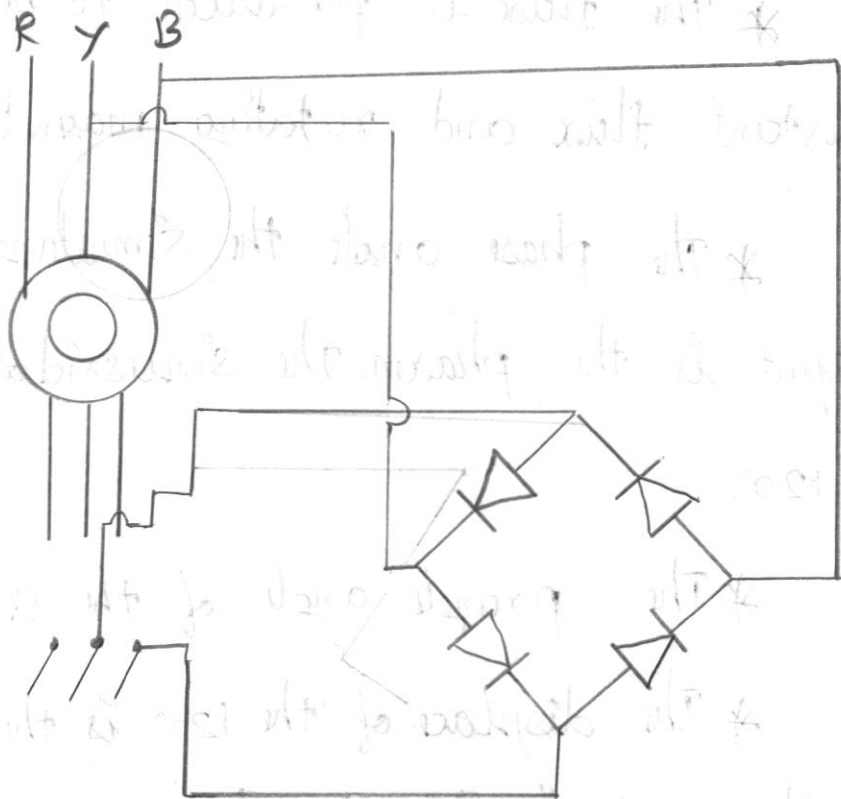
* The DC dynamic Braking is the two terminal is the connected in DC source, one terminal connected in the shorted circuit.

* The three phase supply give to the two terminal is connected or that is called two terminal.

* Third terminal is connected in directly are shorted circuit that is called third terminal.

* The bridge diode are used in the dynamic braking.

* The kinetic energy is stored in the bridge circuit.



* If it act as a generator the kinetic energy is stored and the act as motor the dynamic braking.

* There is the braking is occurs that is called DC dynamic braking.

* The dynamic is the moving of the working the directly supply that is called DC dynamic braking.

15b. Protection of Magnetic field

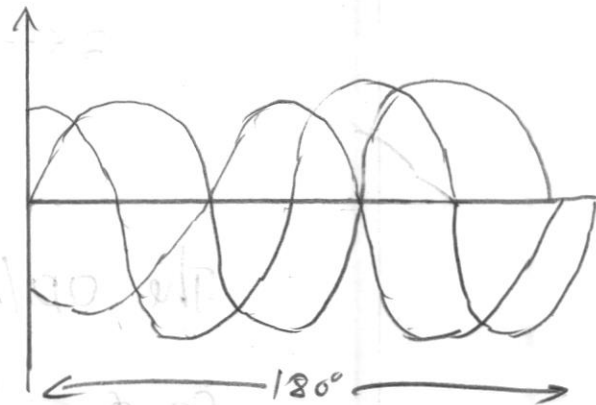
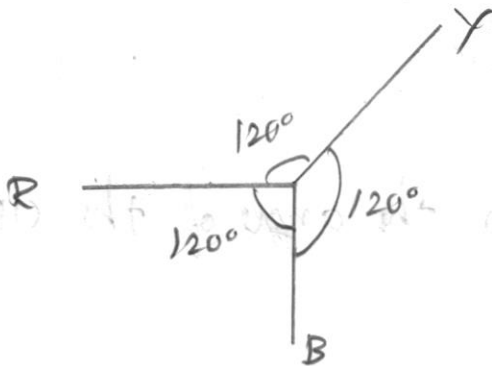
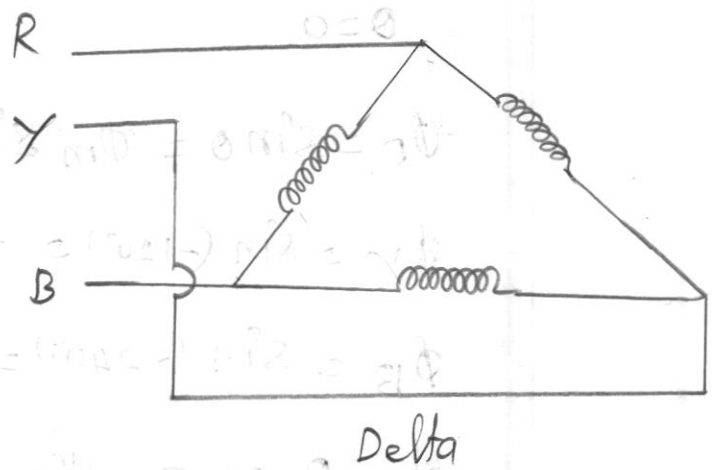
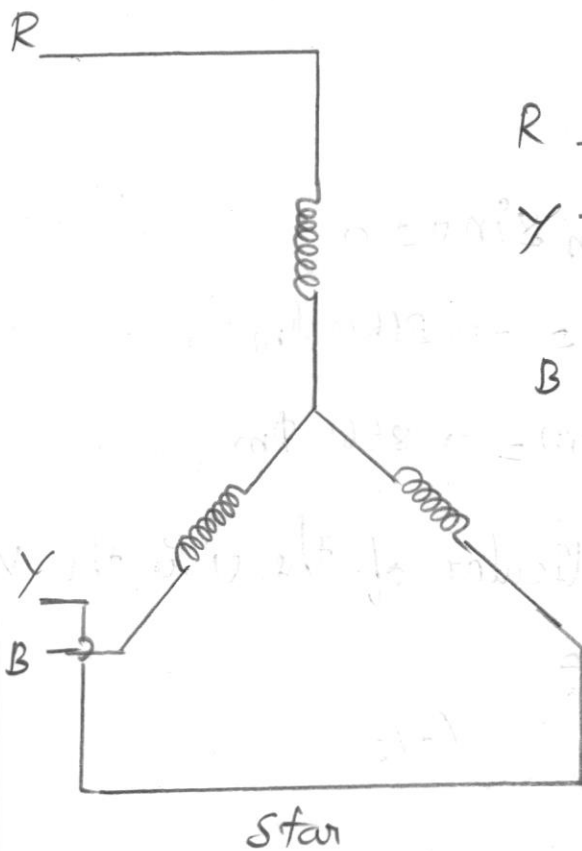
* The three phase induction the 3 ϕ supply is given by the motor.

* The flux is produced to be a the constant flux and rotating magnetic field.

* The phase angle the simultaneously the output is the phase. The sinusoidal displacement is 120° .

* The phase angle of the ϕ_R, ϕ_Y, ϕ_B

* The displace of the 120° is the R, Y, B is the angle is ϕ_R, ϕ_Y, ϕ_B



* The displacement is 180° and angle is the 120° .

$$\phi_R = \phi_m \sin(\omega t - \theta) = \phi_m \sin \theta$$

$$\phi_Y = \phi_m \sin(\omega t - 120^\circ) = \phi_m \sin(\theta - 120^\circ)$$

$$\phi_B = \phi_m \sin(\omega t - 240^\circ) = \phi_m \sin(\theta - 240^\circ)$$

* the find sin values

Case (i)

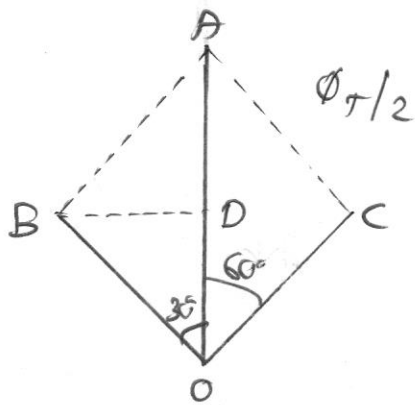
$$\theta = 0$$

$$\phi_R = \sin \theta = \phi_m \sin 0 = 0$$

$$\phi_Y = \sin (-120^\circ) = -0.8660 \phi_m$$

$$\phi_B = \sin (-240^\circ) = 0.8660 \phi_m$$

The B perpendicular of the ϕ' is the value



The OD/AD is the angle of the OB is

$$\cos \phi = \frac{OB}{BD}$$

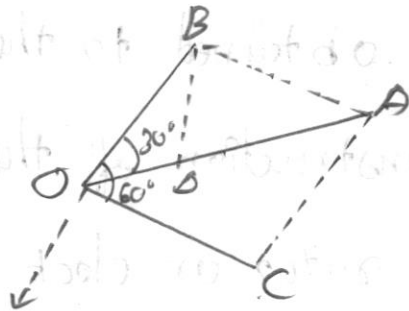
$$\cos \phi = \frac{\phi_T}{2} BD$$

$$\phi_T = 1.5 \phi_T$$

* The value of the $1.5 \phi_T$ is the angle of the value

Case (ii).

$$\phi = 60^\circ$$



* The same construction of the $\phi 60^\circ$ clock wise direction

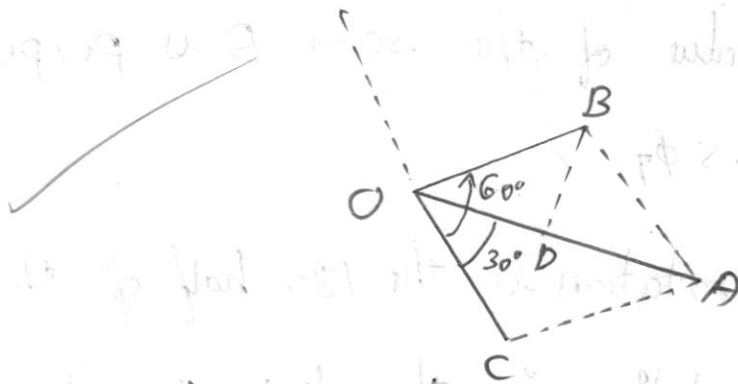
$$\phi_m = \sin 60^\circ = 0.8660$$

$$\phi_m = \sin(60 - 120) = -0.8660$$

$$\phi_m = \sin(60 - 240) = 0$$

* The B is perpendicular of the OB the value of ϕ_T is $1.5 \phi_T$ then same value is clock wise direction.

Case (iii) $\phi = 120^\circ$



The same construction of the $\phi 120^\circ$ is the rotation of the 120° Angle

$$\phi_m = \sin 120^\circ = 0.8660$$

$$\phi_m = \sin 0 = 0$$

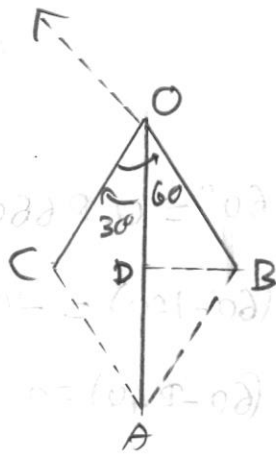
$$\Phi_m = \sin(120 - 240) = -0.2660$$

* The B is perpendicular of the OD

* The value is obtained to the ΔOBA

* The same construction of the conductor the value is $1.5 \Phi_T$ if it is rotor on clock wise direction.

Case $\Phi = 180^\circ$



The same construction of the

$$\Phi_m = \sin 180^\circ = 0$$

$$\Phi_m = \sin(180 - 120) = 0.2660$$

$$\Phi_m = \sin(180 - 240) = -0.2660$$

* The value of the same B is perpendicular value is $1.5 \Phi_T$

* The rotation is the 180° half of the circle that is rotating is the clock wise direction.

Part-c16a. Given data

$$W = 14.71 \text{ kW}, V_2 = 400 \text{ V}, f = 50 \text{ Hz}$$

$$V_0 = 400 \text{ V}, I_0 = 9, \cos \phi_0 = 0.2$$

$$V_{sc} = 200 \text{ V}, I_{sc} = 50 \text{ A}, \cos \phi_{sc} = 0.4$$

$$\text{The open circuit } \phi_0 = \cos^{-1} 0.2$$

$$\phi_0 = 78.46$$

$$\text{The short circuit } \phi_{sc} = \cos^{-1} 0.4$$

$$\phi_{sc} = 66.42$$

The short circuit current I_{SN}

$$I_{SN} = I_{sc} \left(\frac{V_L}{V_{sc}} \right)$$

$$= 50 \left(\frac{400}{200} \right)$$

$$I_{SN} = 100 \text{ A}$$

The short circuit power W_{SN}

$$W_{SN} = W_{sc} \left(\frac{I_{SN}}{I_{sc}} \right)^2$$

The reference axis y axis is the horizontal
is drawn

$$I_0 = 9 \text{ A}$$

Drawn the 1.8 cm

Horizontal axis to the 0 reference line.

* The OO' to the join.

* The ϕ_{sc} to measure 100A is the 5cm scale to draw line.

* The line OA' is the angle.

* Then the OA' is the output line

* The W_{SN} is the

$$W_{SN} = \sqrt{3} V_L I_{SN} \cos \phi_{sc}$$

$$= \sqrt{3} \times 400 \times 100 \times 0.4$$

$$W_{SN} = 27712.8W$$

$$= \frac{22212.8}{8}$$

8

$$W_{SN} = 3464.10W$$

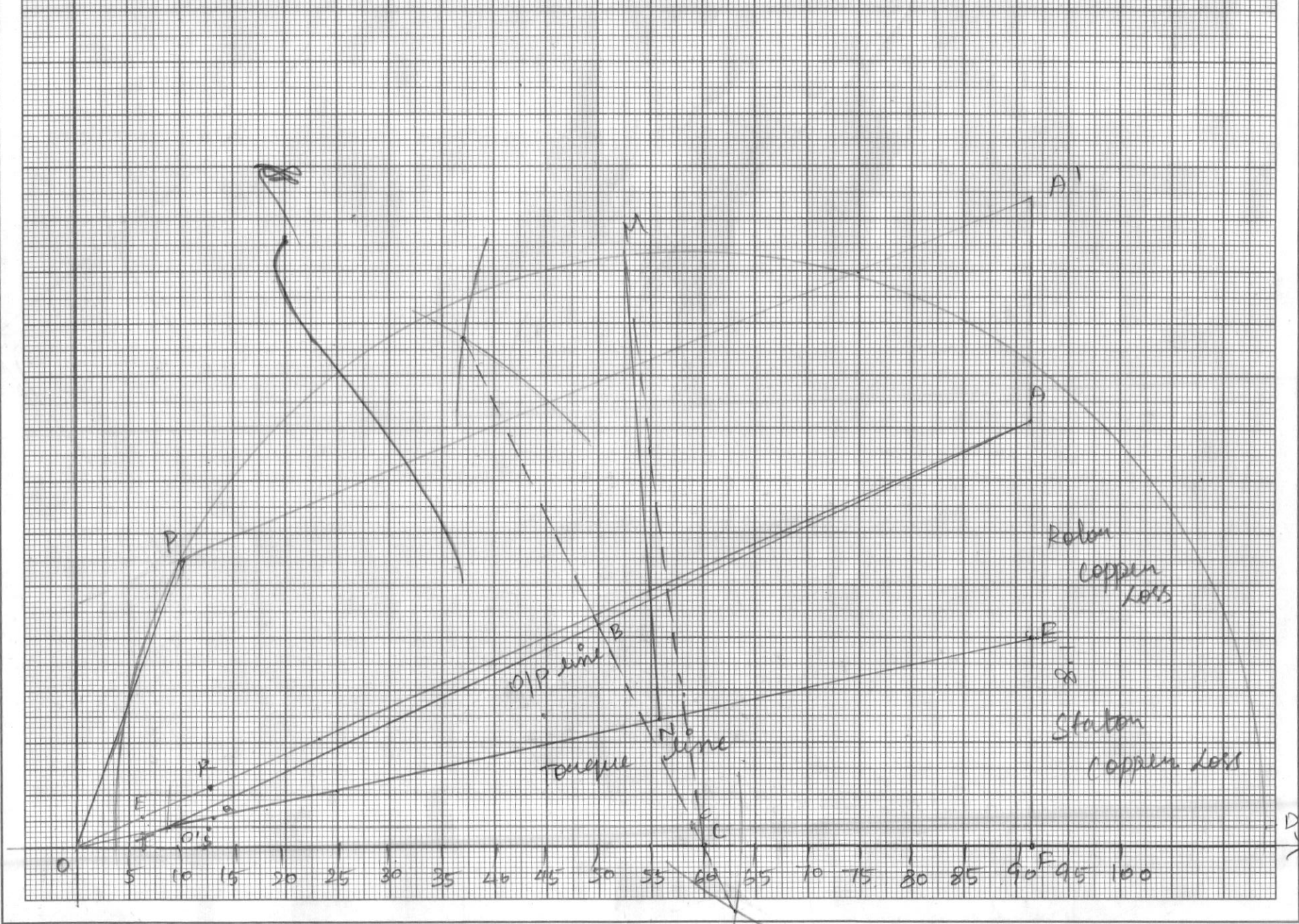
* The measurement of W_{SN} is the rated power

$$= \frac{14.71 \times 10^3}{3464.10} = 4.2 \text{ cm}$$

$$3464.10 = 102W$$

* The AA' to the perpendicular

* Then drawn the semi circle.



ARUN X

* mid point 6

* OA' parallel to the line drawn in P line

* P is join to OP

* The AF is the stator copper loss and rotor copper loss

$$AE = EE$$

Load current is $I_L = 6/2 = 6.2 \text{ cm}$

i. The line current $= 6.2 \times 5 = 31 \text{ A}$

ii. Power factor $\phi = \frac{OP}{PT} = \frac{5.6}{2.4} = 2.3 \approx 0.23$

iii. Slip $= \frac{PA}{AR} = \frac{5.6}{6.2} = 0.056 = 5.6\%$

$$\text{Efficiency} = \frac{PR}{RT} = \frac{5.6}{6.7} \times 100$$

$$= 0.83$$

$$\% \eta = 83.5\%$$

Part - A

1. Standstill Reactance of induction motor.

At standstill, the rotor of the supply frequency is.

$$\text{Rotor frequency } f_r = \text{slip} * \text{stator frequency} \\ = s * f_s$$

$$X_r = 2\pi * (s * f_s) L$$

$$\text{slip } s = 1$$

$$X_r = 2\pi * f_s * L$$

X_r is the rotor reactance of the induction motor. The supply speed is varied with the rotor reactance.

2. Equivalent circuit to the mechanical power developed. The resistance in the circuit model is

$$R_L = R_2 \left(\frac{1}{s} - 1 \right)$$

3. Gen:-

$$\text{Pole} = 4$$

$$f = 50 \text{ Hz}$$

Speed ?

4. Merits of double cage induction motor.

* High starting torque

* Excellent running performance.

5. Demerits of double cage induction motor

Such motor are particularly useful to the larger and when heavy load is

5. Induction motor will never run at its synchronous speed.

* In rotor and stator have same speed.

* Slip zero, no current

* Torque zero etc.

6. Methods used in starting squirrel cage induction motor.

* Auto transformer starter

* Star delta starter

* Delta starter.

7.

Induction Motor

DC Shunt Motor

* The speed control is done by frequency

* The speed control is done by armature control and field control

* It is not self starting

* It is self starting

* No upper speed is required

* The high speed is obtained.

8. Advantages of rotor resistance starter.

* Starting torque can be improved.

* High line current.

* Smooth and wide range of speed control is possible

9. Types of braking.

* Mechanical braking

* Electrical braking

* Regenerative braking

* Plugging

* DC dynamic braking.

10. Advantages of slip power recovery scheme.

* The overall efficiency is improved.

* slip power can be fed back to the supply.



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Continuous Internal Assessment	<u>II</u>						Unit Test					
Register Number	4	2	2	1	1	8	1	0	5	0	2	0
Department	EEE						Semester					
Subject Code	EE8401			Subject Title			Electrical machines-II					
Date & Session	8.3.2020 / FN.						No. of Pages used					

<u>N. K. MATH</u>	<u>N. W. S. / 3 / 2020</u>
Name of the Hall Superintendent	Signature of the Hall Superintendent

Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box											
PART - A			PART - B & C								
Q.No.	✓	Marks	Q.No.	i		ii		iii		Total Marks	
				✓	Marks	✓	Marks	✓	Marks		
			T		D		T		D		
1	✓	2	11	a	✓	2					13
2	✓	2		b							
3	✓	2	12	a							13
4	✓	2		b	✓	13					
5	✓	2	13	a	✓	2	✓	6			13
6	✓	2		b							
7	✓	2	14	a							13
8	✓	2		b	✓	2	✓	6			
9	✓	2	15	a							13
10	✓	2		b	✓	13					
Total			16	a	✓	4					14
				b							
			Total								79
Grand Total	97		Grand Total (in words)								

Name of the Examiner	Signature of the Examiner
----------------------	---------------------------

97

V. Good

PART-B.

11.
a)

Construction of circle diagram:

By the given data obtained by the form of

(i) No load test

(ii) Blocked rotor test

(iii) Stator resistance test

STEP 1:

Test the reference voltage V as phasor (Y axis)

STEP 2:

Select suitable amount such a diameter of circle is about 20 to 30 cm.

STEP 3:

From no load test, I_0 and ϕ_0 are calculated, find ϕ_0 lagging by V as ϕ_0 . This is OO' line.

STEP 4:

The horizontal of DO' is extremely to draw a line parallel to X-axis.

Step 5:

Find I_{SN} is calculated, by the V_o same side of lagging ν by ϕ_{sc} . The OA is produced.

Step 6:

Join $O'A$ in the diagram. This is output line.

Step 7:

Draw the perpendicular bisector of the $O'A$ meets the O' and extend the B . The center of the circle is C .

Step 8:

Draw the semicircle. $O'C$ is the radius of the semicircle. C is the centre of the circle.

Step 9:

Draw the perpendicular line AA' meets the point D . Midpoint of AD is E .

Step 10:

Torque line.

Located at point 'E'

The stator and rotor cu loss of the torque line is plotted on the E'.

The current I_1 and I_2 is produced.

$$k = \frac{I_1}{I_2} \therefore \text{Transformer ratio.}$$

$$\frac{AE}{EF} = \frac{\text{Rotor cu loss}}{\text{Stator cu loss}}$$

$$= \frac{R_2 I_2^2}{R_1 I_1^2} = \frac{R_2}{R_1} \left(\frac{I_2}{I_1} \right)^2$$

$$= \frac{R_2}{R_1} \left(\frac{1}{k} \right)^2$$

Predicted performance of the circle diagram.

The performance of the circle diagram is to be concluded to the power angle of point P.

The squirrel cage induction motor stator

$$\text{cu loss} = 3 I_{SN}^2 \cos \phi.$$

$$W_{SN} = \text{stator cu loss} + \text{rotor cu loss}$$

$$\text{Rotor cu loss} = W_{SN} - \text{stator cu loss}$$

$$= W_{SN} - 3 I_{SN}^2 \cos \phi.$$

Predicted performance:

$$\text{Power factor} = \frac{P_T}{D_P}$$

$$\text{Stator Cu loss} = S_R \times \text{Power scale}$$

$$\text{Rotor Cu loss} = R_R \times \text{Power scale.}$$

$$\text{Total loss} = P_T \times \text{Power scale.}$$

$$\text{Motor efficiency} = \frac{P_Q}{P_T}$$

$$\text{Rotor efficiency} = \frac{P_Q}{R_R}$$

Maximum quantities:

* Maximum current

* Maximum output

* Maximum power factor

* Maximum torque.

* Starting torque.

12)

Torque-slip characteristics:

b)

The curve is plotted torque against slip from $s=1$ (at start) to $s=0$ (synchronous speed). This is called torque-slip characteristics

$$T \propto \frac{SE_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N.m.}$$

$\therefore E_2$ is constant: b.T

$$T \propto \frac{SR_2}{R_2^2 + (sX_2)^2}$$

It has two types of region.

* Low slip region.

* High slip region.

Low slip region:

In low slip region. Due to this

term $(sX_2)^2$ is very very small compared to R_2 . It will be neglected

$$T \propto \frac{SR_2^2}{R_2^2}$$

$T \propto s$.

In low slip region, load increases
load demand increases and speed
decreases.

* The torque is directly proportional to the slip.

$$T \propto s$$

* As the torque increases further the slip increases.

* The torque increases, slip increases and speed decreases.

* The load increases and load demand also increases it satisfy the condition.

* At $s=0$ and $N=N_s$, to the torque if it is tries to attain the synchronous speed.

High slip region:

In high slip region (R_2) is so small compared to the $(sX_2)^2$.

It will be neglected.

$$T \propto \frac{s}{(sX_2)^2}$$

$$T \propto 1/s$$

* The torque is inversely proportional to the slip.

* The torque is increased and the slip is decreased.

* Now, the torque increases, slip increases and speed decreases.

* The speed decreases, slip increases load increases.

* The torque must increased will attain the load demand.

* The torque will drops.

* Even all extra effect on load is added.

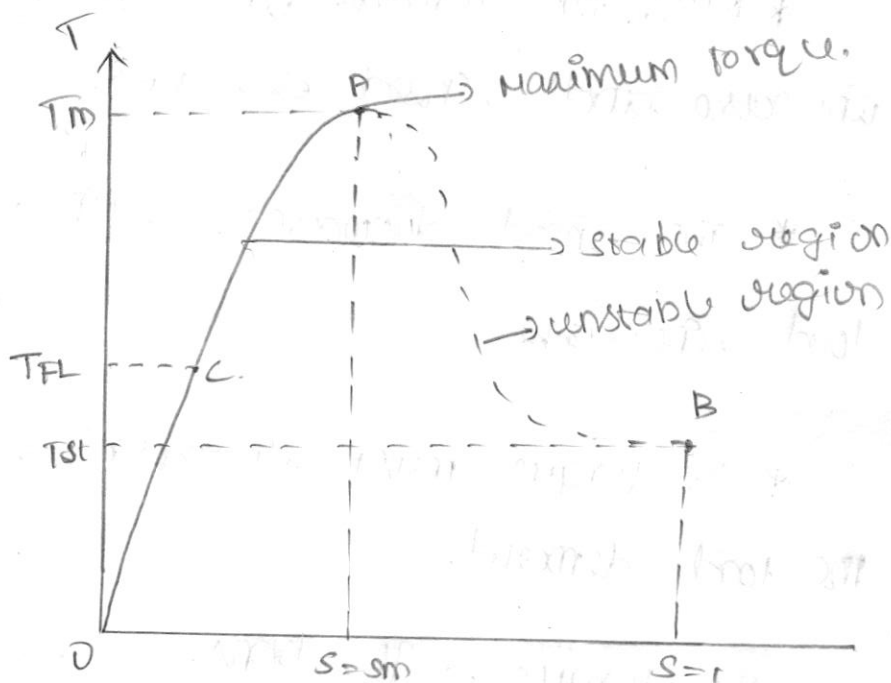
* It satisfy the load demand.

* Then the load increases, torque increases and load demand also increased. It satisfy the condition.

So torque - slip characteristic has two parts.

* straight line called stable region.

* Rectangular hyperbola called unstable region.



OA - stable region.

AB - unstable region.

A (T_m) - maximum torque.

C (T_{FL}) - full load torque.

B (T_{st}) - starting torque.

In the low slip region of the condition satisfy the synchronous speed.

* In the load demand condition is obtain,
by the extra effect of load is added.

It is null out torque

13) a) i) Torque under running condition of a
3 phase induction motor.

In the case of the DC motor
of the torque is directly proportional
to the product of flux per pole and
armature current.

$$T \propto \phi I_a$$

The torque of the 3 phase
induction motor under running condition
is

$$T \propto \phi I_{2r} \cos \phi_{2r}$$

ϕ - flux produced induced emf

I_{2r} - rotor current under running
condition.

$\cos \phi_{2r}$ - Power factor of motor running
condition.

Let,

E_2 is the case of induced emf per
phase under standstill condition.

X_2 is the case of rotor reactance
per phase under standstill condition.

If the slip of the frequency is
obtained,

$$X_{2r} = sX_2$$

$$\therefore E_2 = \phi$$

$$E_{2r} = sE_2$$

If

$$I_{2r} = \frac{E_{2r}}{Z_{2r}}$$

$$I_{2r} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\cos \phi_{2r} = \frac{R_2}{I_{2r}}$$

$$\cos \phi_{2r} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

The torque under running condition.

$$T \propto \phi I_{2r} \cos \phi_{2r}$$

$$T \propto E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\therefore E_2 = \phi$$

$$T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$T = \frac{K s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

K = constant of proportionality

$$K = \frac{.3}{2\pi N_s}$$

$$N_s = \text{synchronous speed} = \frac{N_s}{60}$$

condition for maximum torque:

$$T = \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

The torque is differentiate due to s .

$$\frac{dT}{ds}$$

$$\frac{dT}{ds} = (R_2^2 + s^2 X_2^2) (K E_2^2 R_2) - (K s E_2^2 R_2) (2s X_2^2)$$

$$R_2 = s X_2$$

$$s m = \frac{R_2}{X_2}$$

It is slip from the maximum torque.

$$\text{If slip } s = 1$$

$$T = \frac{K E_2^2 R_2}{(2 X_2)^2}$$

* Maximum torque of the independent rotor resistance (R_2)

* If the induced emf of the under running condition at standstill (E_2)

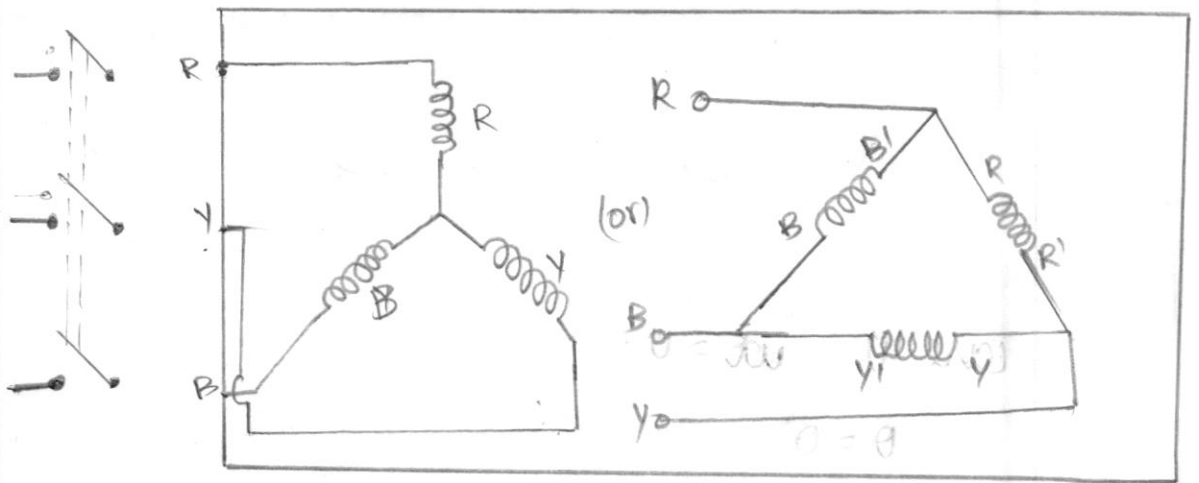
* The maximum torque of the rotor reactance (X_2).

13) a
(ii)

Rotating Magnetic field of 3 ϕ induction motor.

* The working principle of 3 phase induction motor is Rotating Magnetic field.

* The 3-phase induction motor of the R, Y and B is connected.



* The 3 phase induction motor is connected to star (or) delta winding.

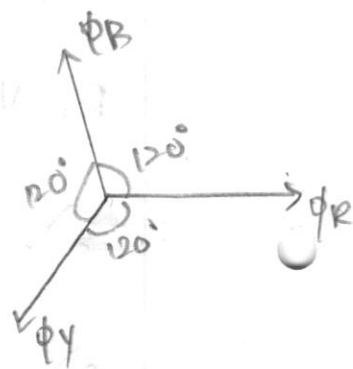
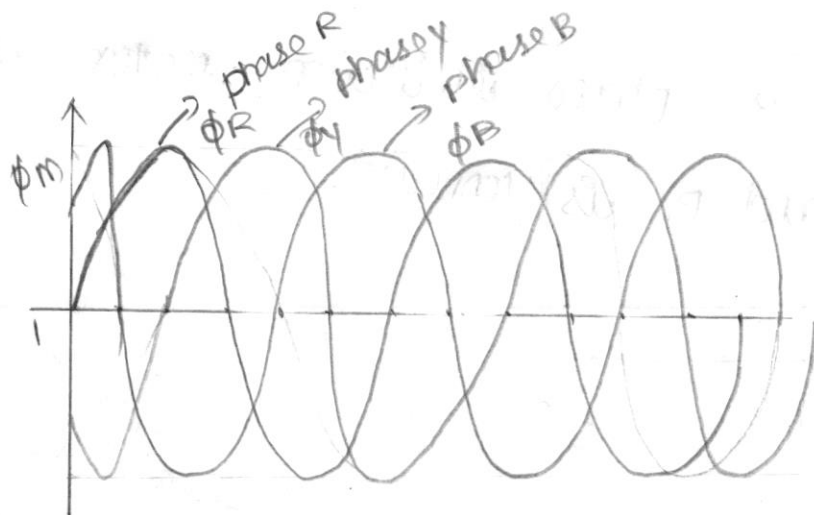
* The three lead connection is three terminal is connected in ac supply.

* Two lead connection is two terminals is connected in ac supply. other one is kept open (or) short circuited.

$$\phi_R = \phi_m \sin \omega t \rightarrow \textcircled{1}$$

$$\phi_Y = \phi_m \sin (\omega t - 120^\circ) \rightarrow \textcircled{2}$$

$$\phi_B = \phi_m \sin (\omega t - 240^\circ) \rightarrow \textcircled{3}$$



Case (i) $\omega t = 0$.

$$\theta = 0^\circ$$

$$\phi_R = \phi_m \sin \theta$$

$$\phi_Y = \phi_m \sin (\theta - 120^\circ)$$

$$\phi_B = \phi_m \sin (\theta - 240^\circ)$$

$$\phi_R = \phi_m \sin 0 = 0$$

$$\phi_Y = \phi_m \sin (-120^\circ) = -0.866 \phi_m$$

$$\phi_B = \phi_m \sin (-240^\circ) = 0.866 \phi_m$$

Draw perpendicular B on angle ϕ_T

In $\triangle OBD$ $\angle BOD = 30^\circ$

$$\cos 30^\circ = \frac{OD}{OB}$$

$$\cos 30^\circ = \frac{\phi_T}{0.866 \phi_m}$$

$$\phi_T = 0.866 \phi_m \times \cos 30^\circ$$

$$\phi_T = 1.5 \phi_m.$$

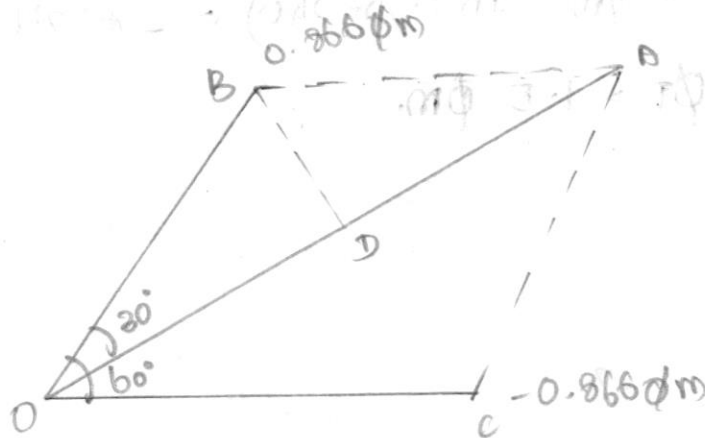
case (ii)

$$\theta = 60^\circ$$

$$\phi_R = \phi_m \sin 60^\circ = 0.866 \phi_m$$

$$\phi_Y = \phi_m \sin(60^\circ - 120^\circ) = -0.866 \phi_m$$

$$\phi_B = \phi_m \sin(60^\circ - 240^\circ) = 0.$$



$$\phi_T = 1.5 \phi_m.$$

Case (iii)

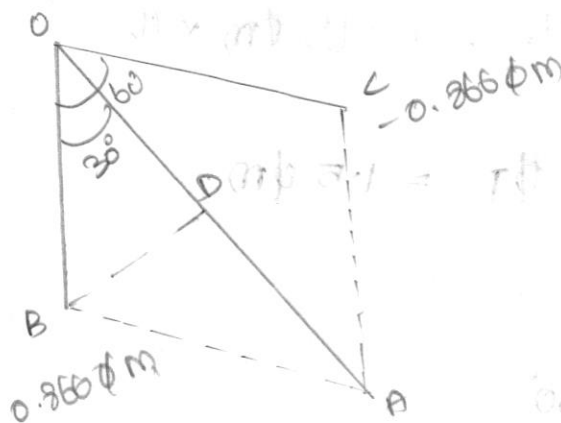
$$\theta = 120^\circ$$

$$\phi_R = \phi_m \sin 120^\circ = 0.866 \phi_m$$

$$\phi_Y = \phi_m \sin(120^\circ - 120^\circ) = 0$$

$$\phi_B = \phi_m \sin(120^\circ - 240^\circ) = -0.866 \phi_m$$

$$\phi_T = 1.5 \phi_m$$



Case (iv)

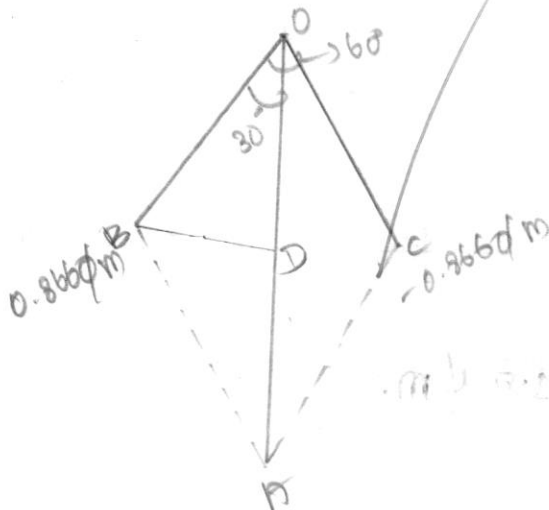
$$\theta = 180^\circ$$

$$\phi_R = \phi_m \sin(180^\circ) = 0$$

$$\phi_Y = \phi_m \sin(180 - 120) = 0.866 \phi_m$$

$$\phi_B = \phi_m \sin(180 - 240) = -0.866 \phi_m$$

$$\phi_T = 1.5 \phi_m$$



* The angle of the vector diagram is each side is increased by 120° electrical.

* The vector diagram is changed clockwise direction.

15)

b)

Given:

$$P_{00} = 6$$

$$f = 50 \text{ Hz}$$

$$T_{FL} = 160$$

$$f' = \frac{120}{60} = 2 \text{ Hz}$$

$$T_{loss} = 10 \text{ Nm}$$

$$\text{stator loss} = 800 \text{ W}$$

$$N_s = \frac{120f}{P} = \left(\frac{120 \times 50}{6} \right) = 1000 \text{ rpm}$$

$$s = \frac{f'}{f} = \frac{2}{50} = 0.04 \%$$

$$N = N_s (1 - s)$$

$$= 1000 (1 - 0.04)$$

$$= 960 \text{ rpm}$$

$$T_{FL} = \frac{P_{out} \times 60}{2\pi N}$$

$$P_{out} = \frac{T_{FL} \times 2\pi N}{60}$$

$$= \frac{160 \times 2\pi \times 960}{60}$$

$$P_{out} = 16085 \text{ W}$$

$$\text{Gross torque} = T_{FL} + T_{loss}$$

$$= 160 + 10$$

$$T_g = 170 \text{ N-m}$$

$$T_g = \frac{P_m \times 60}{2\pi N}$$

$$P_m = \frac{T_g \times 2\pi N}{60}$$

$$= \frac{170 \times 2\pi \times 960}{60}$$

$$= 17090.2 \text{ W}$$

$$P_m = 17 \text{ kW}$$

$$\text{Rotor cu loss} = P_m \left(\frac{s}{1-s} \right)$$

$$s_{2b} = 17000 \left(\frac{0.04}{1-0.04} \right)$$

$$= 17000 \left(\frac{0.04}{0.96} \right)$$

$$= 17000(0.04)$$

$$\text{Rotor cu loss} = 708.3$$

$$\text{Rotor input } P_2 = \left(\frac{P_m}{1-s} \right)$$

$$= \frac{17000}{(1-0.04)}$$

$$P_2 = 17708.$$

$$\text{input power } P_{in} = P_2 + P_{SL}$$

$$= 17708 + 800$$

$$= 18508$$

$$\text{Efficiency } \eta = \frac{P_{out}}{P_{in}} \times 100.$$

$$= \frac{16085}{18508} \times 100$$

$$= 87.1.$$

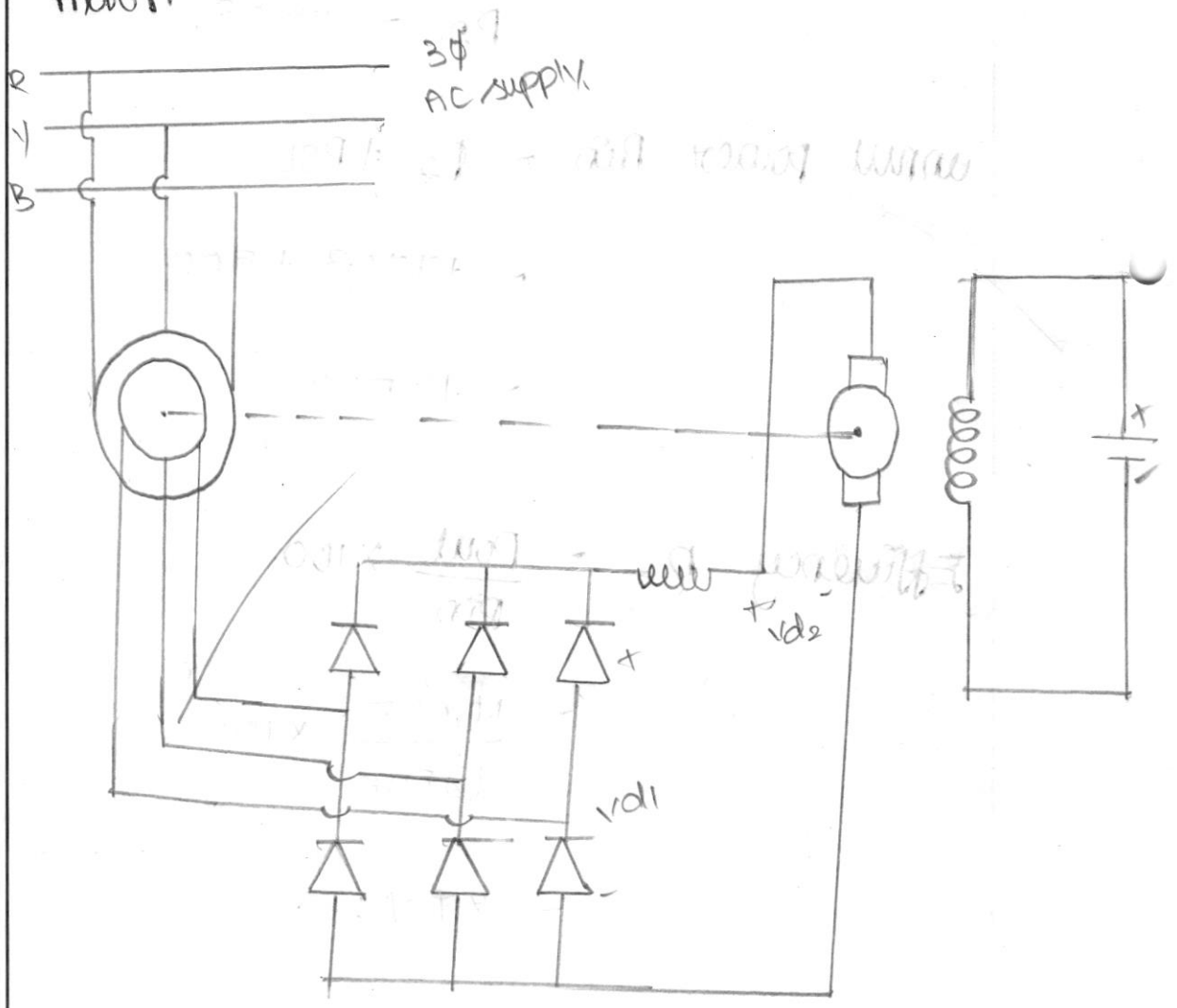
14)
b) i

static Kramer variable speed drive.

* The static Kramer variable speed drive is the method of construction of the 3-phase induction motor.

* By injecting the opposite voltage of the speed and resistance of the machine

* The injecting is changed the speed and resistance is varied by the induction motor.



* The static Kramer variable speed device of the slip power is converted to the AC power.

* When the Kramer drive is not converted to the mechanical power.

* The static Kramer variable speed drive of air gap between the stator and rotor.

* The stator and rotor of the speed will be varied.

* The slip power will get wasted power.

* The Kramer drive is fed back to the wasted power into the main supply.

* It is applicable for the speed of the synchronous motor is low.

* It is applied to the static Kramer variable speed drive.

Advantages!

- * Smooth speed control is possible.
- * wide range of speed control is possible.
- * In the design of rotary converter of speed is maintained.

14)
b (ii)

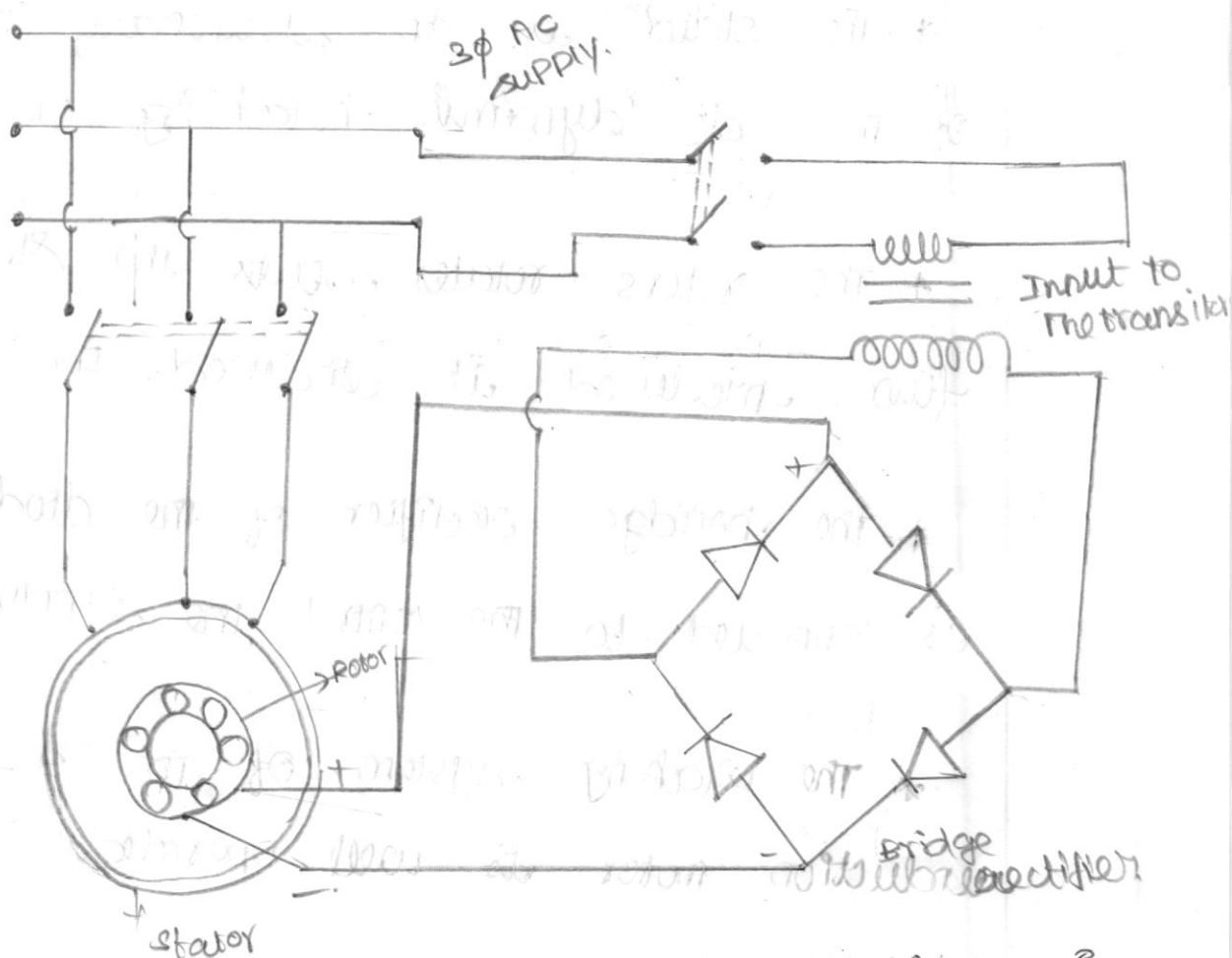
DC Dynamic braking:

* The quick stopping of induction motor is applied in the DC generator dynamic braking.

* The two lead connection is applied in the two terminal in ac supply.

* The other terminal is kept open (or) short circuited.

* The three lead connections are the three terminals are connected in AC supply. It is called, Three lead connection.



* In the three lead connection is three phase ac supply

* The input to the connection transistor is connected two lead,

* The stator and rotor is given to the supply and connected the terminals in the rotor and stator.

* The diode of dc bridge rectifier is connected near the transformer.

* The stator is an stationary part of the dc dynamic braking operation.

* The rotor rotates, cuts the stator flux produced it induced the emf.

* The bridge rectifier of the diode is connected to the control the supply.

* The braking system of the 3-phase induction motor is well operated.

* The three phase induction motor is not self-starting motor.

* The speed is not high of the motor.

* The induction motor is controlled by the frequency.

* It is operated in three phase and to the diode of the bridge rectifier

* In dc dynamic braking of the three phase induction motor is performed well.

Advantages:

* The heat is reduced compared to plugging.

* Quick stopping induction motor.

PART-A.

1) standstill reactance of induction motor.

At standstill, the rotor of the supply frequency is,

$$\begin{aligned} \text{Rotor frequency } f_r &= s * \text{stator frequency} \\ &= s * f_s \end{aligned}$$

$$X_r = 2\pi * (s * f_s) L$$

$$\text{slip } s = 1$$

$$X_r = 2\pi * f_s * L$$

X_r is the rotor reactance of the induction motor. The supply speed is varied with the rotor reactance.

2) Equivalent circuit to the mechanical power developed. The resistance in the circuit model is

$$R_L = R_2 \left(\frac{1}{s} - 1 \right)$$

3) Given :-

$$P_{\text{pole}} = 4$$

$$f = 50 \text{ Hz.}$$

$$\text{speed} = ?$$

$$N_s = \frac{120 f}{P}$$

$$= \frac{120 \times 50}{4}$$

$$= 1500$$

$$N_s = 1500 \text{ rpm.}$$

4) Merits of double cage induction motor.

* High starting torque.

* Excellent running performance.

Demerits of double cage induction motor:

Such motor are particularly useful to the largest and when heavy load is required.

5) Induction motor will never run at its synchronous speed.

* In rotor and stator have same speed.

* slip zero, no current.

* Torque zero etc.

* This is the induction motor will never run at its synchronous speed.

b) Methods used in starting squirrel cage induction motor:

* Auto transformer stator

* Star delta stator

* DOL stator.

7) Induction Motor

DC Shunt motor

The speed control is done by frequency

The speed control is done by armature control and field control.

It is not self starting

It is self starting.

No stepped speed is required

The high speed is obtained.

8) Advantages of rotor resistance starters:

* Starting torque can be improved

* High line current

* ~~smooth~~ and wide range of speed control is possible

9) Types of braking: (not had on part)

- * Mechanical braking.
- * Electrical braking.
- * Regenerative braking.
- * plugging.
- * DC dynamic braking.

10) Advantages of slip power recovery scheme.

- * The overall efficiency is improved.
- * slip power can be fed back to the supply.

part - c.

16) a) Gm:

$$\text{Power} = 14.71 \text{ kW}$$

$$V = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$\text{No load test} = 400 \text{ V}, 9 \text{ A}, \cos\phi = 0.2$$

$$\text{short circuit test} = 200 \text{ V}, 50 \text{ A}, \cos\phi = 0.4$$

From NO load test:

$$\text{No load current } I_0 = 9 \text{ A}$$

$$\text{No load power factor } \cos \phi_0 = 0.2$$

$$\text{No load phase angle} = \phi_0$$

$$\phi_0 = \cos^{-1} 0.2$$

$$\phi_0 = 78.46^\circ$$

From blocked rotor test:

$$V_{sc} = 200 \text{ V} \quad I_{sc} = 50 \text{ A}$$

$$\text{Power factor } \cos \phi_{sc} = 0.4$$

$$\text{Phase angle } \phi_{sc} = \cos^{-1} 0.4$$

$$\phi_{sc} = 66.42^\circ$$

Apply the NO load test find I_{SN}

is calculated.

$$I_{SN} = I_{sc} \left(\frac{V_L}{V_{sc}} \right)$$

$$= 50 \left(\frac{400}{200} \right)$$

$$I_{SN} = 100 \text{ A}$$

* The short circuit test is applied to find the power WSN;

$$W_{SN} = W_{sc} \left(\frac{I_{SN}}{I_{sc}} \right)^2$$

* Draw the $OD = I_0 = 9A$ (i.e) 1.8 cm with y axis.

* Draw the parallel line to x axis at the O'

* Draw the line $OA = I_{SN} = 100A$ (20 cm) in x axis

* Join $O'A$. This is output line.

* Draw the perpendicular bisector of $O'A$ meets the $O'A$.

* Draw the semi-circle of radius $O'C$.

* The center point is C .

* Then draw the horizontal line of x axis meet the D .

$$AE = EF$$

$$W_{SN} = \sqrt{3} V I_{SN} \cos \phi_{sc}$$

$$= \sqrt{3} \times 400 \times 100 \times \cos(0.4)$$

$$= 69280 \text{ W.}$$

$$W_{SN} = 34640 \text{ W.}$$

* The midpoint is E.

* Join O'E is called torque line.

$$AD = 8 \text{ cm.}$$

$$\text{Power scale} = \frac{W_{SN}}{AD}$$

$$= \frac{34640}{8}$$

$$= 4.3 \text{ W.}$$

* To extend the line A into A'

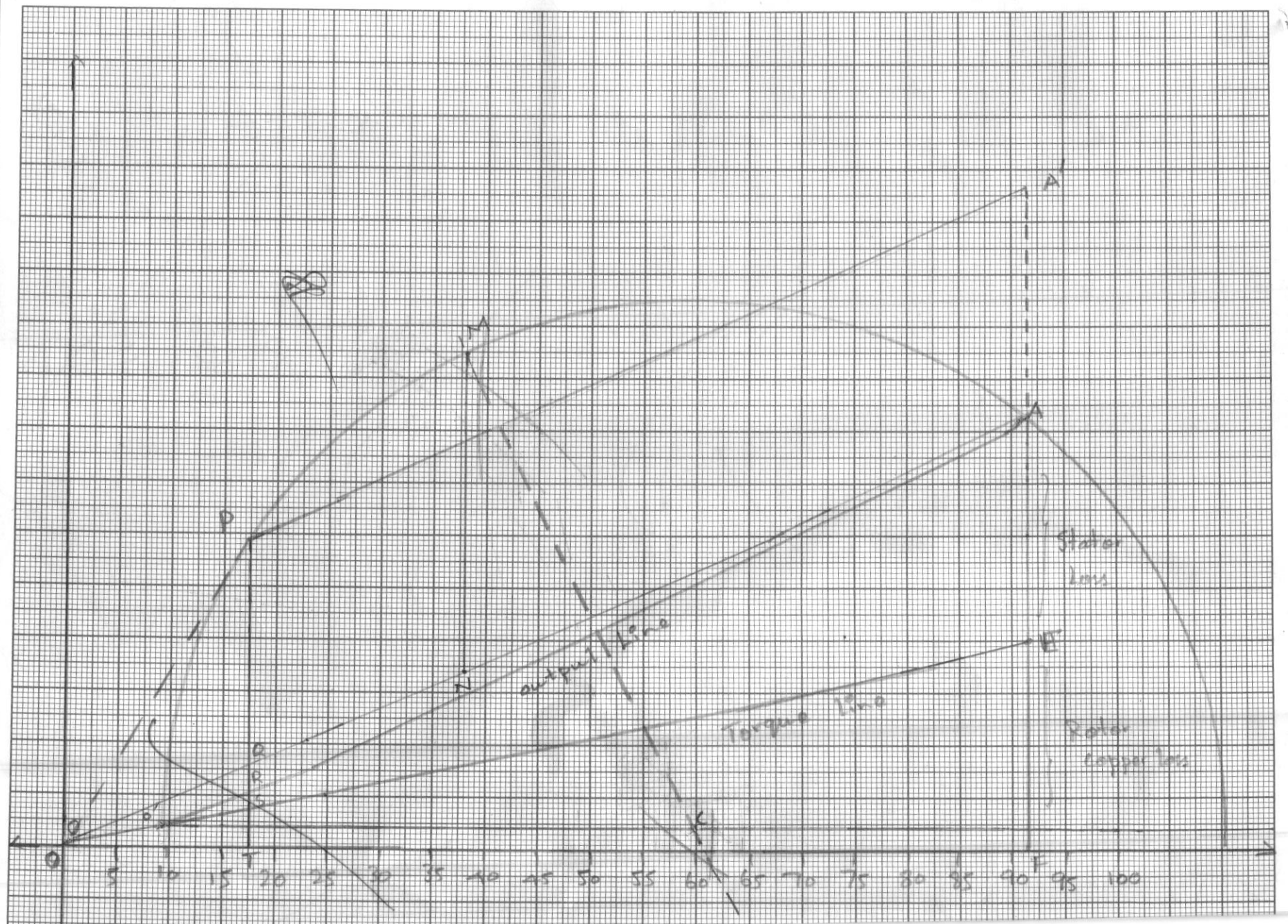
* Draw the parallel line of A is the X axis.

* Join OP of an angle 29°

$$\text{Power} = 14.71 \text{ kW}$$

$$= \frac{14.71 \times 10^{-3}}{34640}$$

$$= 0.42 \text{ cm.}$$



i) line current :

$$\begin{aligned} \text{line current} &= OP = 6 \text{ cm} \quad (1 \text{ cm} = 5 \text{ A}) \\ &= 6 \times 5 \\ &= 30 \text{ A} \end{aligned}$$

(ii) power factor :

$$\cos \phi = \frac{PT}{OP} = \frac{5.2}{6} = 0.861$$

(iii) sfp

$$\text{sfp} = \frac{QR}{PR} = \frac{0.25}{4.3} = 0.0051$$

(iv) Efficiency $\eta = \%$

$$\begin{aligned} &= \frac{PQ}{PT} = \frac{4.2}{5.2} = 0.807 \\ &= 81\% \end{aligned}$$

v) Maximum power output:

In the maximum power output of the region is M, the tangent line is drawn to find MN.

Maximum power output = $l(MN) \times \text{power scale}$

$$= 6.5 \times 34640$$

$$= 225160 \text{ W}$$



ST. ANNE'S

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Continuous Internal Assessment			II					Unit Test			
Register Number	4	2	2	1	1	8	1	0	5	0	7
Department	EEE						Semester				
Subject Code	EE8401		Subject Title		Electrical Machines - II						
Date & Session	3.3.2020/FN						No. of Pages used				

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Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box

PART - A			PART - B & C							Total Marks	
Q.No.	✓	Marks	Q.No.	i		ii		iii			
				✓	Marks	✓	Marks	✓	Marks		
			T D		T D		T D				
1	✓	2	11	a	✓						
2	✓	2		b							
3	✓	2	12	a							
4	✓	1+1		b	✓						
5	✓	2	13	a	✓		✓				
6	✓	2		b							
7	✓	2	14	a							
8	✓	1		b	✓		✓	5x1			
9	✓	1+1	15	a							
10	✓	1+1		b	✓						
Total	16+3		16	a	✓						
				b							
			Total						Total		69
Grand Total			Grand Total (in words)								

Name of the Examiner	A. P. R. S.	Signature of the Examiner	A. P. R. S.
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A. P. R. S.

Part - B

11.

a) Construction of circle diagram:

By the given data obtained by the form of

- i) No load test
- ii) Blocked motor test
- iii) stator resistance test

step 1:

Test the reference voltage V as Phasor (Y axis)

step 2:

Select suitable current such a diameter of circle is about 20 to 30 cm

step 3:

From no load test, I_0 and ϕ_0 are calculated, find ϕ_0 lagging by V as ϕ_0 . This is OO' line.

step 4:

The horizontal of OO' is extremely to draw a line O' parallel to X axis.

Step 5:

Find I_{sc} is calculated, by the
to same side of lagging V by ϕ_{sc} .
The OA is produced.

Step 6:

Join O'A in the diagram

This is output line.

Step 7:

Draw the perpendicular bisector of
the O'A meets the circle and extend the
B. The centre of the circle is C.

Step 8:

Draw the semicircle

O'C is the radius of the semicircle
C is the centre of the circle.

Step 9:

Draw the perpendicular line A
meets the point D. midpoint of AD is E.

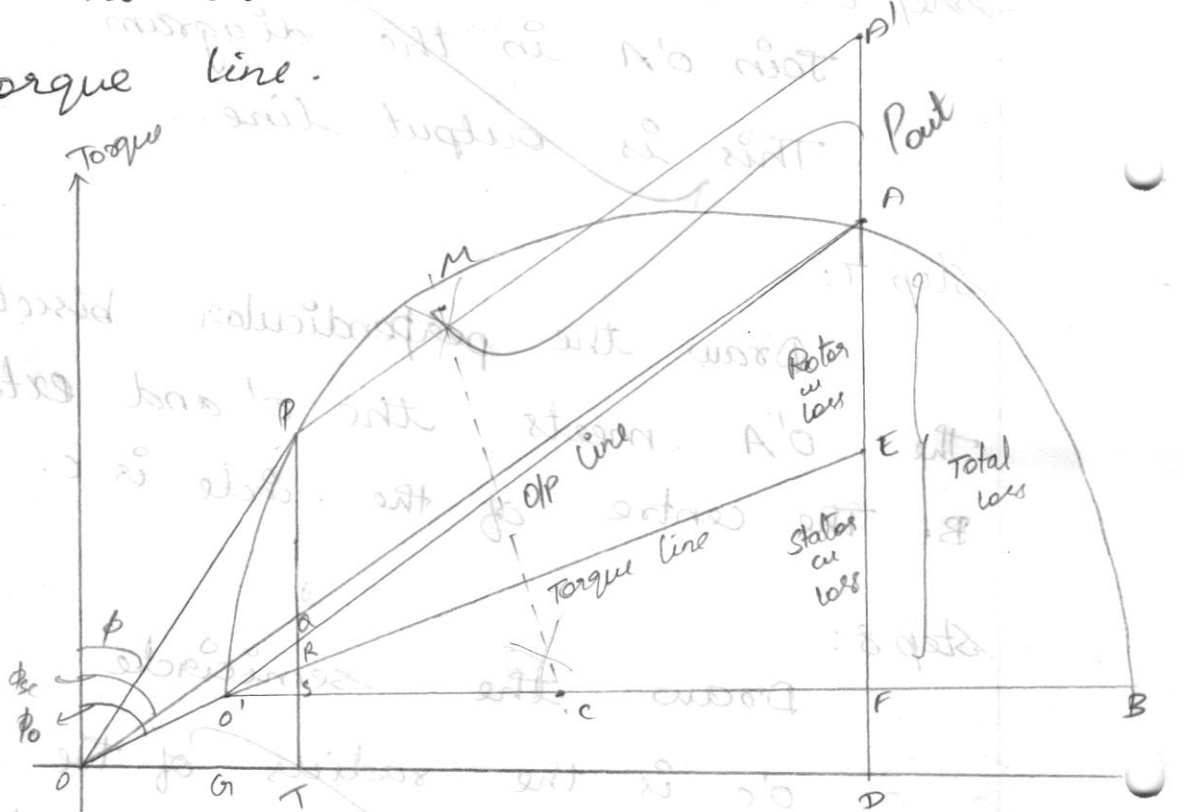
Step 10:

Torque line.

Torque is separated by the core loss and stator, rotor copper loss.

$$\frac{AE}{EP} = \frac{\text{Rotor cu loss}}{\text{stator cu loss}}$$

The O'E line under the condition of torque line.



Power scale:

The power scale can be obtained in the 'AD' calculate W_{SN} of the stator output.

$$\text{Power scale} = l(AD) \times W_{SN} \text{ W/cm}$$

$$= \frac{W_{SN}}{AD}$$

AD is cm

located at point 'E'

The stator and rotor cu loss of the torque line is plotted in the 'E'

The current I_1 and I_2 is produced.

$$K = \frac{I_1}{I_2} \quad \therefore \text{Transformer ratio.}$$

$$\frac{AE}{EF} = \frac{\text{Rotor cu loss}}{\text{Stator cu loss}}$$

$$= \frac{R_2 I_2^2}{R_1 I_1^2} = \frac{R_2}{R_1} \left(\frac{I_2}{I_1} \right)^2$$

$$= \frac{R_2}{R_1} \left(\frac{1}{K^2} \right)$$

Predicted performance of the circle diagram.

The performance of the circle diagram is to be conducted to the power angle of point P.

In squirrel cage induction motor.

$$\text{Stator cu loss} = 3 I_{SN}^2 \cos \phi$$

$$W_{SN} = \text{stator cu loss} + \text{Rotor cu loss}$$

$$\text{Rotor cu loss} = W_{SN} - \text{stator cu loss}$$

$$= W_{SN} - 3 I_{SN}^2 \cos \phi$$

Predicted Performance:

$$\text{Power factor} = \frac{PT}{OP}$$

$$\text{stator cu loss} = SR \times \text{power scale}$$

$$\text{Rotor cu loss} = OR \times \text{power scale}$$

$$\text{Total loss} = PT \times \text{power scale}$$

$$\text{Motor efficiency} = \frac{PO}{PT}$$

$$\text{Rotor efficiency} = \frac{PO}{OR}$$

Maximum quantities:

- * Maximum input
- * Maximum output
- * Maximum power factor
- * maximum torque
- * Starting torque

12.

b) Torque - slip characteristics:

The curve is plotted torque against slip from $s=1$ (at start) to $s=0$ (synchronous speed). This is called torque slip characteristics.

$$T \propto \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2} N - m$$

$\therefore E_2$ is constant

$$T \propto \frac{SR_2}{R_2^2 + (SX_2)^2}$$

It has two types of region

* Low slip region

* High slip region

Low slip region :-

In low slip region, due to this term $(SX_2)^2$ is very very small compared to R_2 . It will be neglected.

$$T \propto \frac{SR_2^2}{R_2^2}$$

$$T \propto S$$

In low slip region, load increases, load demand increase and speed decreases.

→ The torque is directly proportional to the slip

$$T \propto s$$

→ Its the torque increase further the slip increases.

→ The torque increase, slip increases and speed decreases.

→ The load increase and load demand also increase it satisfy the condition

→ At $s=0$ and $N=N_s$, to the torque if it is tries to attain the synchronous speed.

High slip region.

In high slip region (R_2) is so small compared to the $(sX_2)^2$.

It will be neglected.

$$T \propto \frac{s}{(sX_2)^2}$$

$$T \propto \frac{1}{s}$$

→ The torque is inversely

Proportional to the slip.

→ The torque is increased and the slip is decreased.

→ Now, the torque increases, slip increase and speed decreases.

→ The speed decreases, slip increases load increases.

→ The torque must increase will attain the load demand.

→ The torque will drop

→ Eventually extra effort on load is added.

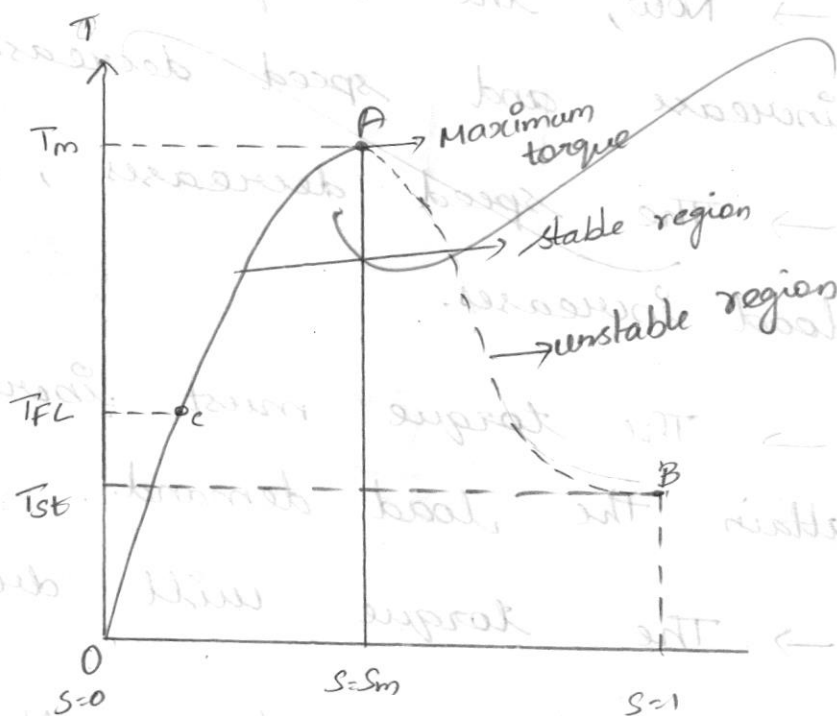
→ It satisfy the load demand.

→ Then the load increases, torque increases and load demand also

increased. It satisfy the condition.

So torque - slip characteristic has two parts.

- * straight line called stable region
- * Rectangular hyperbola called unstable region.



OA - stable region

AB - unstable region

A (T_m) - maximum torque

B (T_{st}) - Full load torque

B (T_{st}) - starting torque

In the low slip region of the condition satisfy the synchronous speed.

In the load demand condition is obtain by the extra effect of load is added. It is pull out torque.

13. a) i) Torque under running condition of a 3 phase induction motor.

In the case of the DC motor of the torque is directly proportional to the product of flux per pole and armature current.

$$T \propto \phi I_a$$

The torque of the 3 phase induction motor under running condition is -

$$T \propto \phi I_{2r} \cos \phi_{2r}$$

ϕ - flux produced induced emf
 I_{2r} - rotor current under running condition.

$\cos \phi_{2r} =$ Power factor of rotor running condition

Let,

E_2 is the case of induced emf per phase under standstill condition

X_2 is the case of rotor reactance per phase under standstill condition

If the slip of the frequency is obtained.

$$X_{2r} = sX_2$$

$$\therefore E_2 = \phi$$

$$E_{2r} = sE_2$$

If

$$I_{2r} = \frac{E_{2r}}{Z_{2r}}$$

$$I_{2r} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}}$$

$$\cos \phi_{2r} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

The torque under running condition

$$T \propto \phi I_{2r} \cos \phi_{2r}$$

$$T \propto E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\therefore E_2 = \phi$$

$$T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$T = \frac{K s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$K =$ constant of proportionality

$$K = \frac{3}{2\pi N_s}$$

$$N_s = \text{synchronous speed} = \frac{N_s}{60}$$

Condition for maximum torque.

$$T = \frac{KE_2^2 R_2}{R_2^2 + (SX_2)^2}$$

The torque is differentiated due to the

$$\frac{dT}{ds}$$

$$\frac{dT}{ds} = (R_2^2 + S^2 X_2^2)(KE_2^2 R_2) - (KE_2^2 R_2)(2SX_2^2)$$

$$R_2 = SX_2$$

$$S_m = \frac{R_2}{X_2}$$

It is slip from the maximum torque

If slip $s=1$

$$T = \frac{KE_2^2 R_2}{(2X_2)^2}$$

* Maximum torque of the independent rotor
Resistance (R_2)

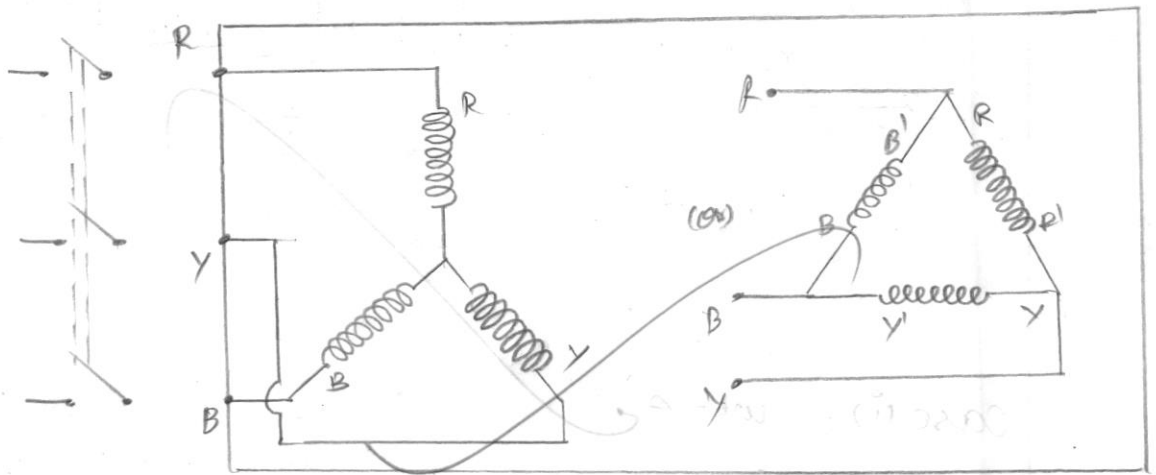
* If the induced emf of the motor
under running conditions at standstill (E_2^2)

* The maximum torque of the motor
reactance (X_2^2).

B.
a) ii)

Rotating magnetic field of 3 ϕ induction motor
The working principle of 3 phase induction motor is Rotating magnetic field.

The 3 phase induction motor of the R, Y and B is connected.



The 3 phase induction motor is connected to star (or) delta winding.

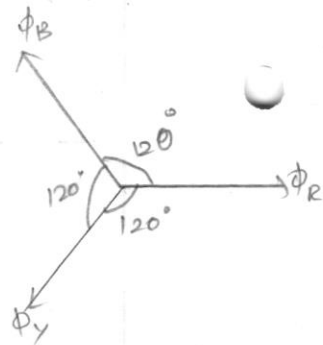
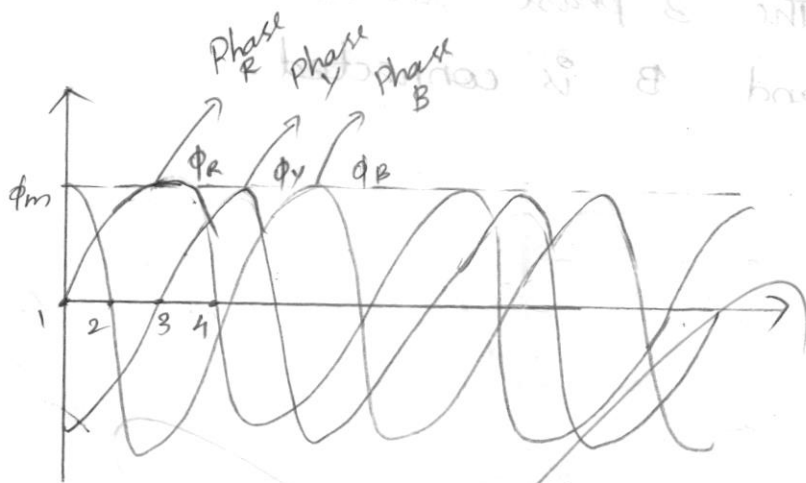
The three lead connection is three terminal is connected in ac supply.

Two lead connection is two terminals is connected in ac supply. other one is kept open (or) short circuited.

$$\phi_R = \phi_m \sin \omega t \quad \text{--- (1)}$$

$$\phi_Y = \phi_m \sin (\omega t - 120^\circ) \quad \text{--- (2)}$$

$$\phi_B = \phi_m \sin (\omega t - 240^\circ) \quad \text{--- (3)}$$



Case (i) $\omega t = \theta$

$$\theta = 0^\circ$$

$$\phi_R = \phi_m \sin \theta$$

$$\phi_Y = \phi_m \sin (\theta - 120^\circ)$$

$$\phi_B = \phi_m \sin (\theta - 240^\circ)$$

$$\phi_R = \phi_m \sin 0 = 0$$

$$\phi_Y = \phi_m \sin (-120^\circ) = -0.866 \phi_m$$

$$\phi_B = \phi_m \sin (-240^\circ) = 0.866 \phi_m$$

Draw perpendicular B an
angle ϕ_T

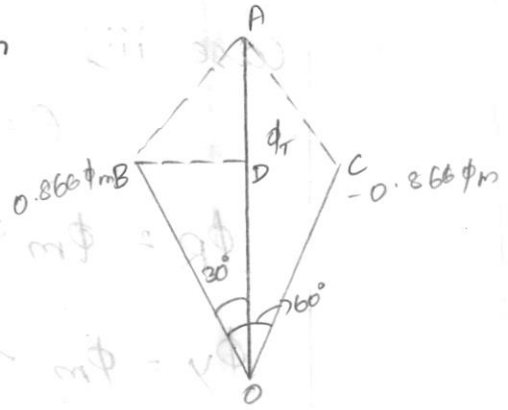
$$\text{In } \triangle OBD \quad \angle BOD = 30^\circ$$

$$\cos 30^\circ = \frac{OD}{OB}$$

$$\cos 30^\circ = \frac{\phi_T}{0.866\phi_m}$$

$$\phi_T = 0.866\phi_m \times \cos 30^\circ$$

$$\phi_T = 1.5\phi_m$$



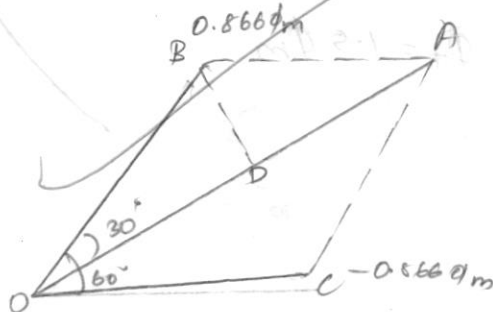
case (ii)

$$\theta = 60^\circ$$

$$\phi_R = \phi_m \sin 60^\circ = 0.866\phi_m$$

$$\phi_y = \phi_m \sin(60 - 120) = -0.866\phi_m$$

$$\phi_B = \phi_m \sin(60 - 240) = 0$$



$$\phi_T = 1.5\phi_m$$

Case iii)

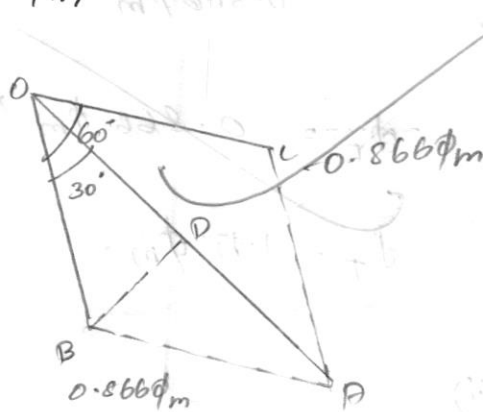
$$\theta = 120^\circ$$

$$\phi_R = \phi_m \sin 120^\circ = 0.866 \phi_m$$

$$\phi_y = \phi_m \sin (120^\circ - 120^\circ) = 0$$

$$\phi_B = \phi_m \sin (120^\circ - 240^\circ) = -0.866 \phi_m$$

$$\phi_T = 1.5 \phi_m$$



Case iv)

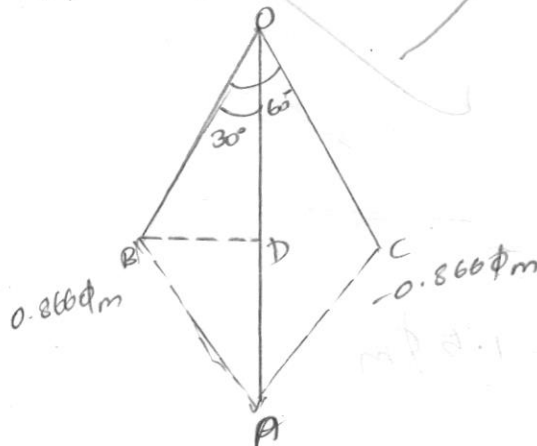
$$\theta = 180^\circ$$

$$\phi_R = \phi_m \sin (180^\circ) = 0$$

$$\phi_y = \phi_m \sin (180^\circ - 120^\circ) = 0.866 \phi_m$$

$$\phi_B = \phi_m \sin (180^\circ - 240^\circ) = -0.866 \phi_m$$

$$\phi_T = 1.5 \phi_m$$



→ The angle of the vector diagram is each side is increased by 120° electrical.

→ The vector diagram is changed clockwise direction.

15.

b)

Gen:-

$$\text{Pole} = 6$$

$$f = 50 \text{ Hz}$$

$$T_{FL} = 160$$

$$f' = 120/60 = 2 \text{ Hz}$$

$$T_{\text{loss}} = 10 \text{ N}\cdot\text{m}$$

$$\text{Stator Loss} = 800 \text{ W}$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$s = \frac{f'}{f} = \frac{2}{50} = 0.04\%$$

$$N = N_s (1 - s)$$

$$= 1000 (1 - 0.04)$$

$$= 960 \text{ rpm}$$

$$T_{FL} = \frac{P_{out} \times 60}{2\pi N}$$

$$P_{out} = \frac{T_{FL} \times 2\pi N}{60}$$
$$= \frac{160 \times 2\pi \times 960}{60}$$

$$P_{out} = 16085 \text{ W}$$

$$\text{Gross torque} = T_{FL} + T_{Loss}$$
$$= 160 + 10$$

$$T_g = 170 \text{ N}\cdot\text{m}$$

$$T_g = \frac{P_m \times 60}{2\pi N}$$

$$P_m = \frac{T_g \times 2\pi N}{60}$$

$$= \frac{170 \times 2\pi \times 960}{60}$$

$$P_m = 17090.2 \text{ W} = 17 \text{ kW}$$

$$\text{Rotor cu loss} = P_m \left(\frac{s}{1-s} \right)$$

$$= 17000 \left(\frac{0.04}{1-0.04} \right)$$

$$= 17000 \left(\frac{0.04}{0.96} \right)$$

$$= 17000 (0.04)$$

$$\text{Rotor Cu loss} = 708.3 \text{ W}$$

$$\text{Rotor input } P_2 = \left(\frac{P_m}{1-s} \right)$$

$$= \frac{17000}{(1-0.04)}$$

$$P_2 = 17708 \text{ W}$$

$$\text{input power } P_{in} = P_2 + P_{sl}$$

$$= 17708 + 800$$

$$= 18508 \text{ W}$$

$$\text{Efficiency } \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{16085}{18508} \times 100$$

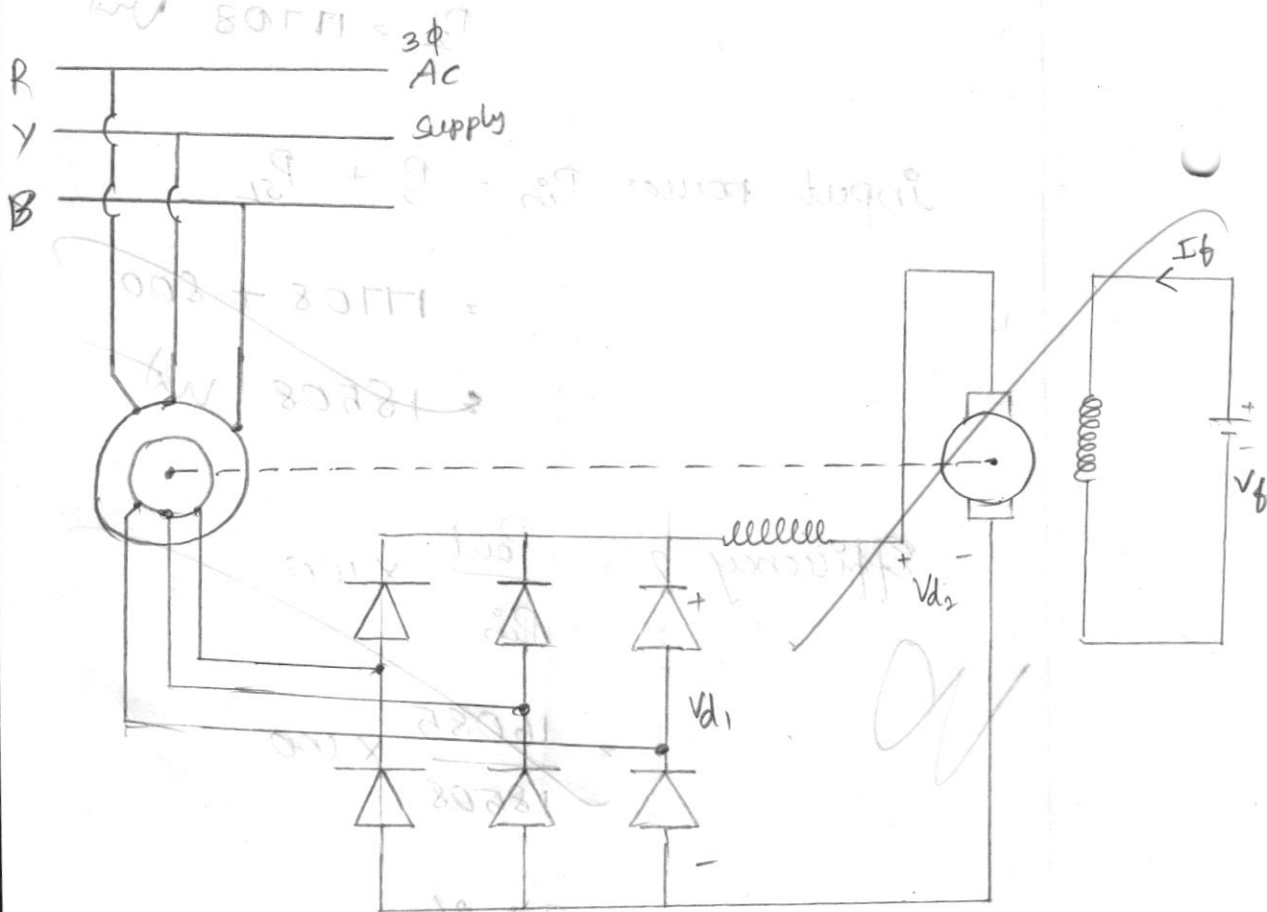
$$= 87 \%$$

14.
b) i) Static Kramer variable speed drive

* The static Kramer variable speed drive is the method of construction of the 3 phase induction motor.

* By injecting the opposite voltage of the speed and resistance of the machine.

* The injecting is changed the speed and resistance is varied by the induction motor.



* The static Kramer Variable speed drive of the slip power is converted to the AC power

* when the Kramer drive is not converted to the mechanical power

* The static Kramer variable speed drive of air gap between the stator and rotor.

* The stator and rotor of the speed will be varied.

* The slip power will get wasted Power.

* The Kramer drive is fed back to the wasted power into the main supply.

* It is applicable for the speed of the synchronous motor is low.

* It is applied to the static Kramer variable speed drive.

Advantage:

- * Smooth speed control is possible
- * wide range of speed control is

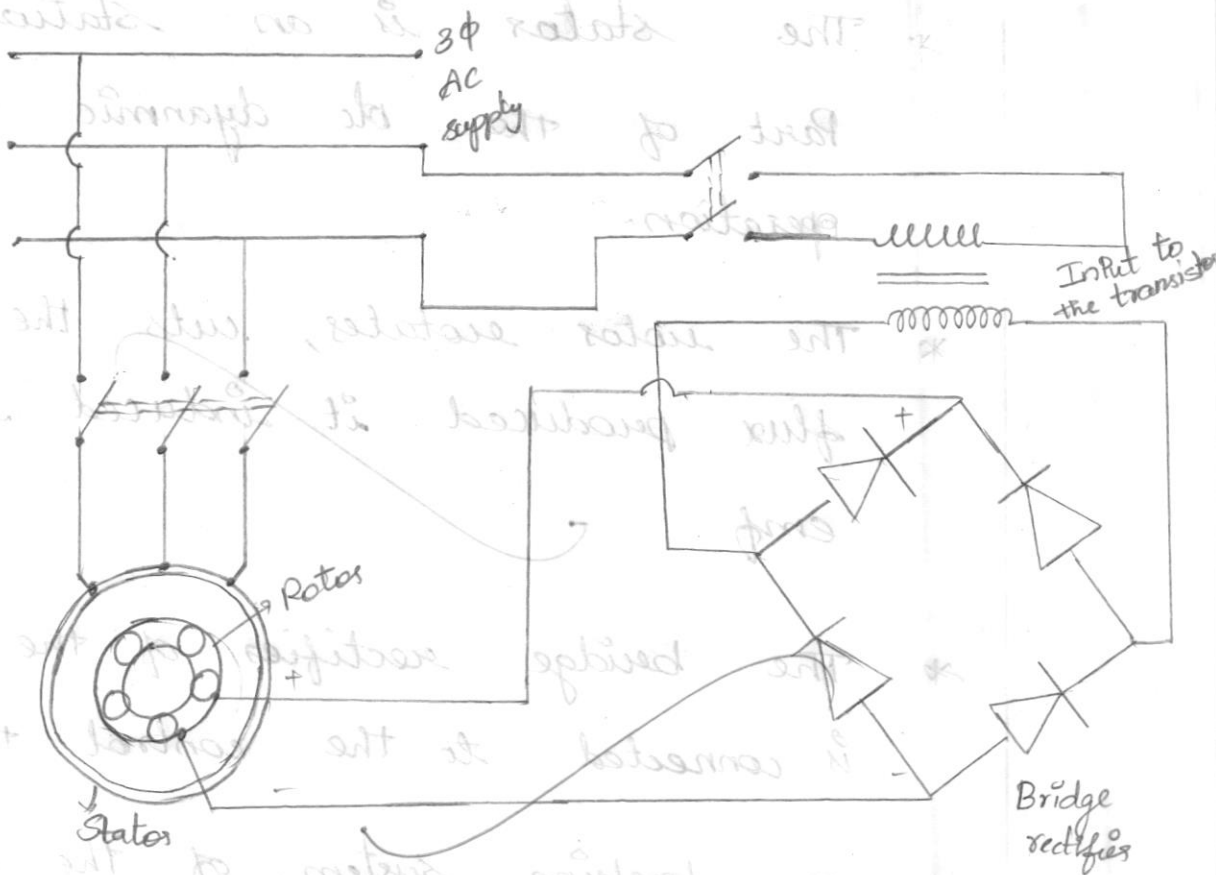
Possible

- * It the design of rotary converter of speed is maintained.

14.
b)ii)

Dc Dynamic braking :-

- * The quick stopping of induction motor is applied in the dc dynamic braking.
- * The two lead connection is applied in the two terminal is ac supply
- * The other terminal is kept open (or) short circuited.
- * The three lead connections are the three terminals are connected in ac supply. It is called three lead connections.



* In the three lead connection
is 3 phase ac supply.

* The input to the transistor is
connected two lead.

* The stator and Rotor is given
to the supply and connected the
terminals in the Rotor and stator.

* The diode of dc bridge rectifies
is connected near the transformer

* The stator is a stationary part of the dc dynamic braking operation.

* The motor rotates, cuts the stator flux produced it induced the emf

* The bridge rectifier of the diode is connected to the control the supply.

* The braking system of the 3 phase induction motor is well operated.

* The 3 phase induction motor is not self starting motor.

* The speed is not high of the motor.

* The induction motor is controlled by the frequency.

* It is operated in 3 phase and to the diode of the bridge rectifier.

* In dynamic braking of the 3 phase induction motor is performed well.

Advantages :-

* The heat is ~~less~~ reduced compared to plugging

* Quick stopping induction motor

Part - A

1. Standstill Reactance of induction motor:

At standstill, the motor of the supply frequency is,

Rotor frequency $f_r = \text{slip} \times \text{stator frequency}$
 $= s \times f_s$

$$X_{st} = 2\pi \times (s \times f_s) L$$

$$\text{slip } s = 1$$

$$X_r = 2\pi \times f_s \times L$$

X_r is the motor reactance of the induction motor. The supply speed is

varied with the motor reactance.

2. Equivalent circuit to the mechanical power developed. The resistance in the circuit model

is

$$R_L = R_2 \left(\frac{1}{s} - 1 \right)$$

3. Given

$$\text{Pole} = 4$$

$$f = 50 \text{ Hz}$$

speed ?

$$N_s = \frac{120f}{P}$$
$$= \frac{120 \times 50}{4}$$

$$= 1500$$

$$N_s = 1500 \text{ rpm}$$

4. Merits of double cage induction motor

* High starting torque

* Excellent running performance

Demerits of double cage induction motor

Such motor are particularly useful to the larger and when heavy load is required.

5. Induction motor will never run at its synchronous speed.

* In motor and stator have same speed.

* slip zero, no current

* Torque zero etc,

This is the induction motor will never run at its synchronous speed.

6. methods used in starting squirrel cage induction motor:

* Auto transformer starter

* star delta starter

* DOL starter

7.

Induction Motor	Dc Shunt Motor
* The ^{ind} speed control is induction motor is frequency	The speed control is armature

7.

Induction Motor	Dc shunt Motor
* The speed control is done by frequency	* The speed control is done by armature control and field control
* It is not self starting	* It is self starting
* NO supper speed is required	* The high speed is obtained

8.

Advantages of motor resistance starter

- * starting torque can be improved
- * High line current
- * smooth and wide range of speed control is possible

9. Types of braking:

* Mechanical braking

* Electrical braking

* Regenerative braking

* Plugging

* DC dynamic braking

10. Advantages of slip power recovery scheme.

* The overall efficiency is improved

* slip power can be fed back to the supply.

Part - c

Q. 16. a) Gn :-

$$\text{Power} = 14.71 \text{ kW}$$

$$V = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$

No load test: 400 V, 9 A, $\cos \phi = 0.2$

short circuit test: 200 V, 50 A, $\cos \phi = 0.4$

From No load test;

$$\text{NO load current } I_0 = 9 \text{ A}$$

$$\text{NO load power factor } \cos \phi_0 = 0.2$$

No load phase angle, ϕ_0

$$\phi_0 = \cos^{-1} 0.2$$

$$\phi_0 = 78.46^\circ$$

From blocked rotor test;

$$V_{sc} = 200 \text{ V}, I_{sc} = 50 \text{ A}$$

$$\text{Power factor } \cos \phi_{sc} = 0.4$$

$$\text{Phase angle } \phi_{sc} = \cos^{-1} 0.4$$

$$\phi_{sc} = 66.42^\circ$$

Apply the no load test find I_{SN} is calculated.

$$I_{SN} = I_{sc} \left(\frac{V_L}{V_{sc}} \right)$$

$$= 50 \left(\frac{400}{200} \right)$$

$$I_{SN} = 100 \text{ A}$$

The short circuit test is applied to find the power W_{SN} :

$$W_{SN} = W_{SC} \left(\frac{I_{SN}}{I_{SC}} \right)^2$$

* Draw the $OO' = I_0 = 9A$ (i.e) 1.8 cm with X axis.

* Draw the parallel line to X axis in the O' .

* Draw the line OA $I_{SN} = 100A$ (20 cm) in X axis.

* Join $O'A$. This is output line

* Draw the perpendicular bisector

of $O'A$ meets the $O'B$.

* Draw the semi circle of radius $O'C$.

* The centre point is e .

* Then draw the horizontal line of X axis meet the D .

$$AE = EF$$

$$\begin{aligned}
 W_{SN} &= \sqrt{3} V I_{SN} \cos \phi_{sc} \\
 &= \sqrt{3} \times 400 \times 100 \times \cos(10.4) \\
 &= 69280 \text{ W} \Rightarrow \frac{W_{SN}}{2} = 34640 \text{ W}
 \end{aligned}$$

* The midpoint is E.

* Join O'E is called torque line.

$$AD = 8 \text{ cm}$$

$$\begin{aligned}
 \text{Power scale} &= \frac{W_{SN}}{AD} \\
 &= \frac{34640}{8}
 \end{aligned}$$

$$= 4.3 \text{ kW}$$

* To extend the line A into A'.

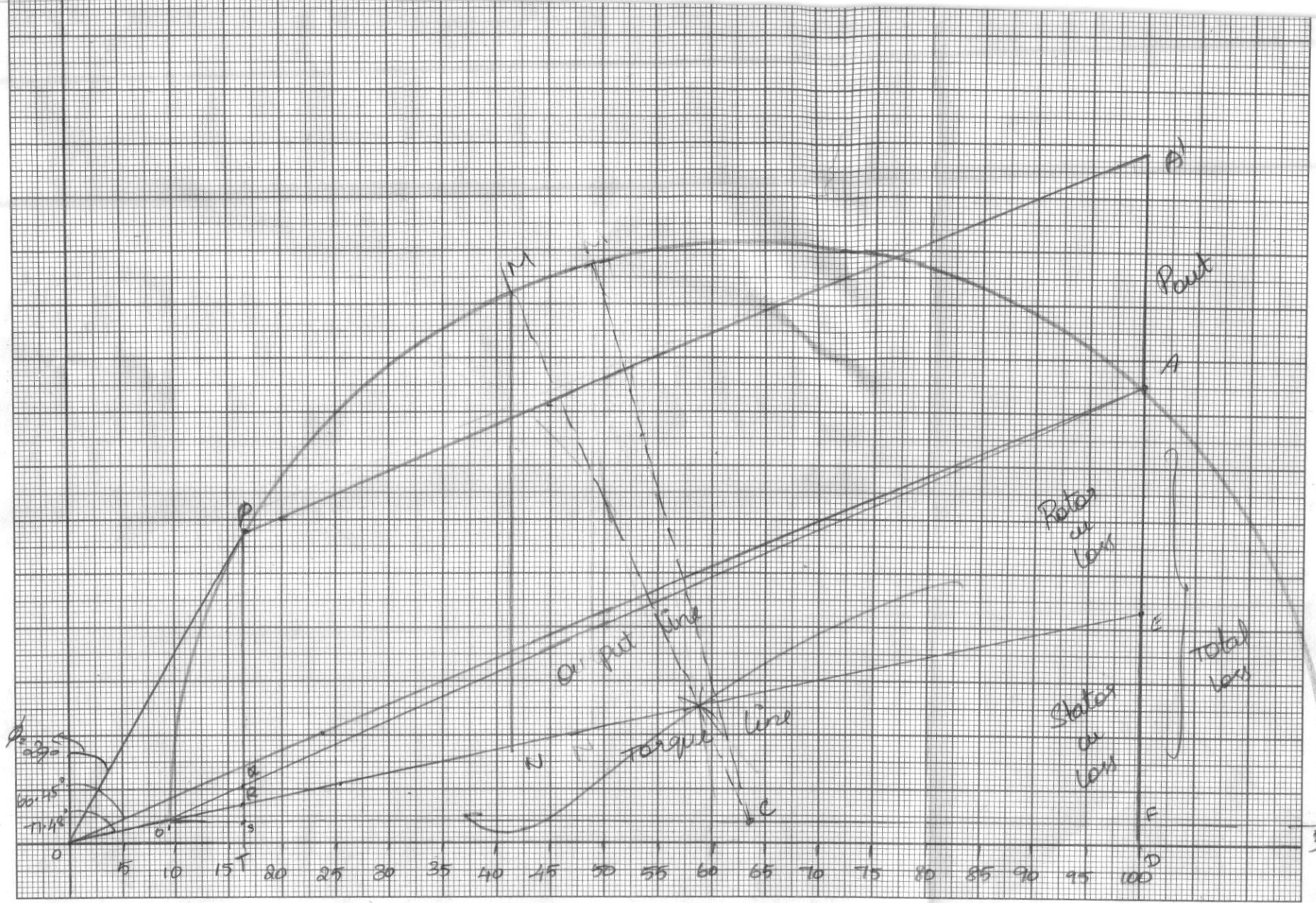
* Draw the parallel line A' of A in the x-axis

* Join op of an angle 29°

$$\text{Power} = 14.71 \text{ kW}$$

$$= \frac{(14.71 \times 10^3)}{34640}$$

$$= 0.42 \text{ cm}$$



i) line current

$$\begin{aligned}\text{line current} &= OP = 6 \text{ cm} && (1 \text{ cm} = 5 \text{ A}) \\ &= 6 \times 5 \\ &= 30 \text{ A}\end{aligned}$$

ii) Power factor

$$\cos \phi = \frac{PT}{OP} = \frac{5.2}{6} = 0.86 \text{ p.u.}$$

iii) slip :-

$$\text{slip} = \frac{QR}{PR} = \frac{0.25}{4.3} = 0.05 \%$$

iv) efficiency $\eta\%$ = $\frac{PQ}{PT} = \frac{4.2}{5.2} = 0.807$
= 81%

v) maximum power output:

In the maximum power output of the region is 'M'. The tangent line is drawn to find 'MN'.

Maximum power output = $l(\text{MN}) \times \text{power scale}$.

$$= 6.5 \times 34640$$

$$= 225160 \text{ W}$$



85

100

~~85

100~~

ST. ANNE'S

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Continuous Internal Assessment				- 1				Unit Test				-																			
Register Number		A		2		2		1		1		8		1		0		6		0		1		7							
Department												ECE												Semester				III			
Subject Code								EC-8851								Subject Title								Electronics circuit - I							
Date & Session												10.08.2019												No. of Pages used				16			

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Name of the Hall Superintendent	Signature of the Hall Superintendent

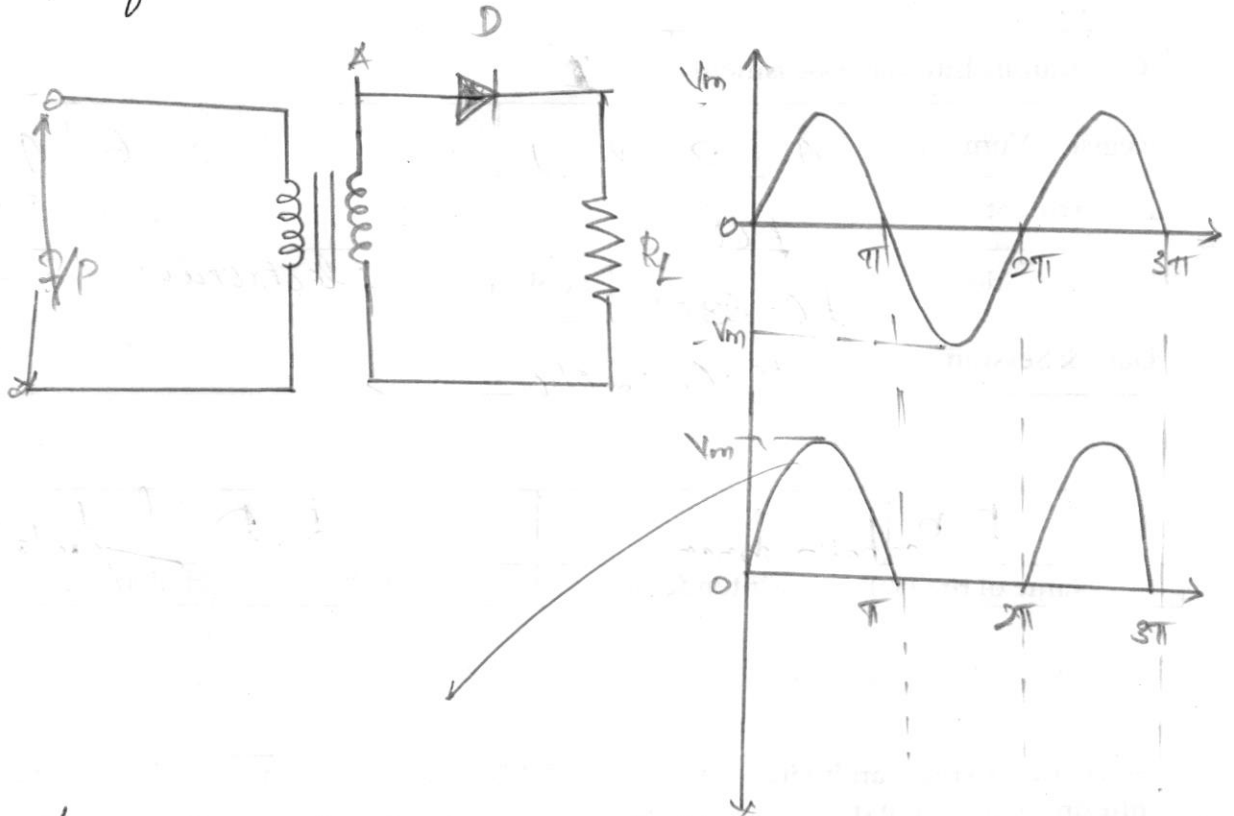
Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box															
PART - A			PART - B & C												
Q.No.	✓	Marks	Q.No.	i		ii		iii		Total Marks					
				✓	Marks	✓	Marks	✓	Marks						
					T	D	T	D	T	D					
1	✓	2	11	a	✓	12								12	
2	✓	2		b											
3	✓	2	12	a											
4	✓	1		b	✓	9								9	
5	✓	2	13	a	✓	9								9	
6	✓	2		b											
7	✓	2	14	a	✓	12								12	
8	✓	2		b											
9	✓	2	15	a											
10	✓	2		b	✓	10								10	
Total			16	a	✓	14								14	
				b											
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Grand Total		84	Grand Total (in words)												
Name of the Examiner				V. VENKATESAN				Signature of the Examiner							

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PART-B

11. a) Half wave rectifier:



Shows the diagram of Half wave rectifier and the I/P & O/P waveforms of HWR.

In Half wave rectifier, the ~~self~~ rectifier conducts only during +ve half cycle of I/P AC supply.

the regenerative half cycle of AC supply are suppressed.

$$V_o = V_m \sin \omega t$$

$$V_o = 0$$

$$I_L = \frac{V_m \sin \omega t}{R_L}$$

$$= I_m \sin \omega t$$

$$I_L = 0$$

$$\text{for } 0 < \omega t < \pi$$

$$\text{for } \pi < \omega t < 2\pi$$

$$\text{for } 0 < \omega t < \pi$$

$$\text{for } \pi < \omega t < 2\pi$$

Average or DC value:

$$I_{av} = I_{dc} = \frac{\text{Area under the wave over full cycle}}{\text{Base}}$$

$$= \frac{1}{2\pi} \int_0^{2\pi} I_L \cdot d\omega t$$

$$= \frac{1}{2} \int_0^{2\pi} I_m \sin \omega t \cdot dt$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} I_m \sin \omega t \cdot d\omega t + \int_{\pi}^{2\pi} I_m \sin \omega t \cdot dt \right]$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} I_m \sin \omega t \cdot d\omega t + 0 \right]$$

$$= \frac{1}{2\pi} \int_0^{\pi} I_m \sin \omega t \cdot d\omega t$$

$$= \frac{I_m}{2\pi} \left[-\cos \omega t \right]_0^{\pi}$$

$$= \frac{-I_m}{2\pi} \left[\cos 2\pi - \cos 0 \right]$$

$$= \frac{-I_m}{2\pi} \left[-1 - 1 \right]$$

$$I_{dc} = \frac{I_m}{\pi}$$

$$V_{dc} = \frac{V_m}{\pi}$$

RMS value of load current (I_{rms}):

$$I_{rms} = \frac{\sqrt{\text{Area under the squared rectifier wave over a cycle}}}{\text{period of base.}}$$

$$\begin{aligned} I_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} I_L^2 \cdot d\omega t} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \omega t)^2 \cdot d\omega t} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{\pi} (I_m \sin \omega t)^2 \cdot d\omega t + \int_{\pi}^{2\pi} (I_m \sin \omega t)^2 \cdot d\omega t} \\ &= \sqrt{\frac{1}{2\pi} \left[\int_0^{\pi} (I_m \sin \omega t)^2 \cdot d\omega t + 0 \right]} \\ &= \sqrt{\frac{I_m^2}{4\pi}} \quad \pi = \sqrt{\frac{I_{rms}^2}{4\pi}} \end{aligned}$$

$$I_{rms} = \frac{I_m}{2}$$

$$V_{rms} = \frac{V_{m}}{2}$$

Ripple factor:

The unwanted AC components present in the DC opp are called ripple factor. Ripple can be removed by using filter circuit.

$$I_{RMS} = \sqrt{I_{dc}^2 + I_1^2 + I_2^2 + \dots}$$

$$= \sqrt{\frac{P_{dc}^2}{I_{dc}^2} + I_{dc}^2} = \sqrt{I_{dc}^2 + \frac{P_{rms}^2}{I_{dc}^2}}$$

$$\gamma = \frac{P_{rms}}{I_{dc}} = \frac{P_{ac}}{I_{dc}}$$

$$= \sqrt{\frac{P_{rms}^2 - I_{dc}^2}{I_{dc}^2}}$$

$$= \sqrt{\left(\frac{P_{rms}}{I_{dc}}\right)^2 - 1}$$

HWR, $P_{rms} = \frac{P_m}{2}$ & $I_{dc} = \frac{P_m}{\pi}$

$$\gamma = \sqrt{\left(\frac{P_m/2}{P_m/\pi}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{\pi}{2}\right)^2 - 1}$$

$$\boxed{\gamma = 1.21}$$

Efficiency:

Rectifier efficiency is defined as the ratio of the DC o/p power (P_{dc}) to the AC i/p power (P_{ac}).

$$\eta = \frac{P_{dc}}{P_{ac}}$$

$$P_{dc} = I_{dc}^2 \cdot R_c \quad P_{ac} = I_{rms}^2 (R_s + R_f + R_L)$$

$R_L =$ Load Resistance

$R_f =$ Forward Resistance of the ckt.

$R_s =$ Secondary resistance of the transformer

$$\eta = \frac{P_{dc} \cdot R_L}{P_{rms} (R_s + R_f + R_L)}$$

$$P_{rms} = \frac{I_m}{2} \text{ \& } P_{dc} = \frac{I_m}{8}$$

$$\eta = \frac{(I_m/n)^2 \cdot R_L}{(I_m/2)^2 (R_s + R_f + R_L)}$$

$$= \frac{4}{n^2} \left(\frac{R_L}{R_s + R_f + R_L} \right)$$

$$= 0.406 \times \frac{R_L}{R_c} \left(\frac{1}{(R_s + R_f) + R_L} \right)$$

$$\eta = 0.406$$

$$\boxed{\eta = 40.6\%}$$

Peak Inverse Voltage: (PIV):

$$\boxed{PIV = V_m}$$

Transformer utilisation factor (TUF)

$$\frac{P_{dc}}{P_{ac \text{ rated}}} = TUF$$

$$P_{ac \text{ rated}} = V_{ac \text{ rated}} \cdot I_{ac \text{ rated}}$$

$$= \frac{V_m}{2} \cdot \frac{I_m}{2}$$

$$P_{dc} = I_{dc}^2 \cdot R_L$$

$$= \left(\frac{I_m}{\pi} \right)^2 \cdot R_L$$

2, 8 in WKT

$$TVP = \left(\frac{I_m}{\pi} \right)^2 \cdot \frac{R_L}{\frac{I_m}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$= \frac{I_m}{\pi^2} \cdot \frac{R_L \cdot 2\sqrt{2}}{I_m^2 \cdot R_L}$$

$$= \frac{2\sqrt{2}}{\pi^2}$$

$$\boxed{TVP = 0.287}$$

Form factor.

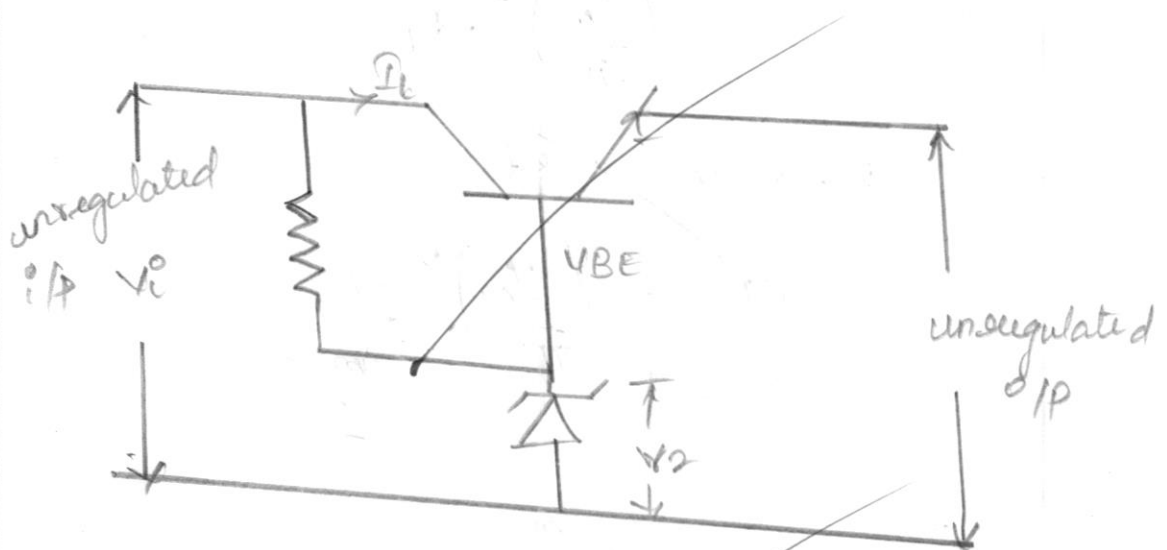
$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

$$= \frac{V_m / \sqrt{2}}{V_m / \pi} = \pi / \sqrt{2} \Rightarrow \boxed{PF = 2}$$

Load current: (I_L):

$$I_L = \frac{I_m}{\pi} + \frac{I_m}{2} \sin \omega t - \frac{2I_m}{8\pi} \cos 2\omega t - \frac{2I_m}{15\pi} \cos \omega t.$$

12) b) series regulator:



Series voltage is called voltage regulator. Since, the transistor is ~~series~~ series with load resistance, this is also called emitter follower.

this is called emitter follower, because its transistor behaves like an emitter.

$$V_{BE} = V_z - V_o$$

$$V_{BE} = V_z - V_o$$

$$V_o = V_z - V_{BE}$$

transistor Q_1 is series control element
 & Zener diode provides reference voltage.

Regulating factor:

$$V_L \downarrow, V_{BE} \uparrow, V_{CE} \downarrow, I_L \uparrow$$

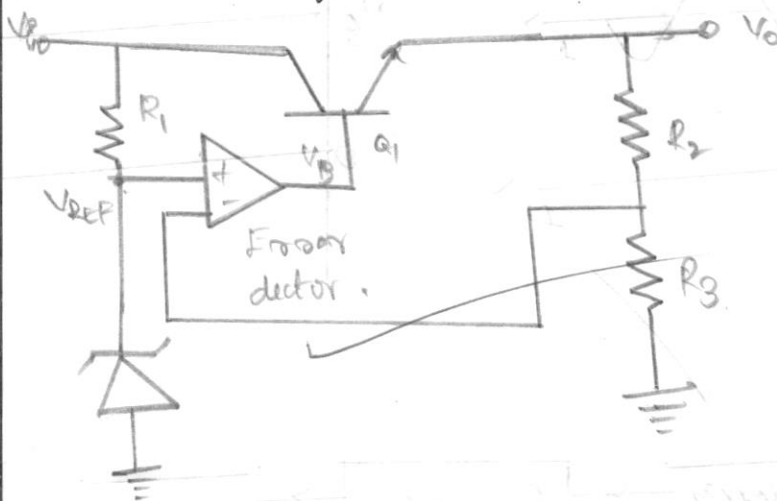
$\therefore V_o \Rightarrow$ maintained constant.

Flow graph:

$$V_L \uparrow, V_{BE} \downarrow, V_{CE} \uparrow$$

$\therefore I_L \downarrow, V_o \Rightarrow$ maintained constant.

op-amp regulated based voltage regulator:-



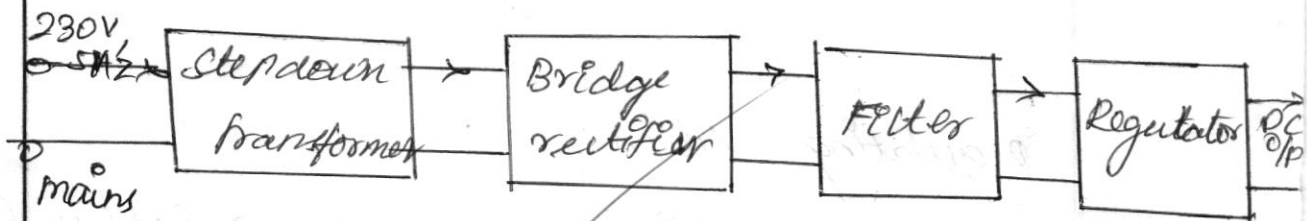
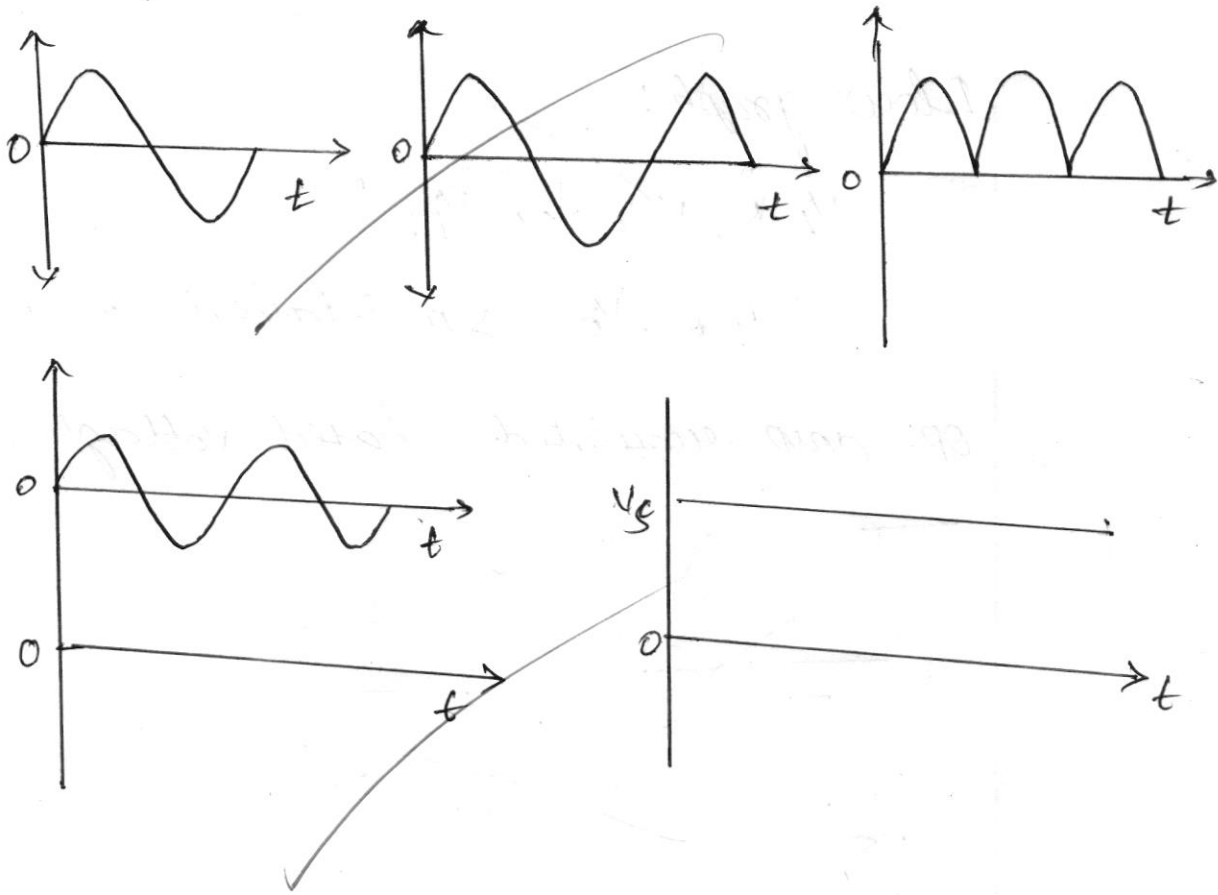
Regulating Action:

the resistive voltage divider formed by R_2 and R_3 sense any change in the op voltage

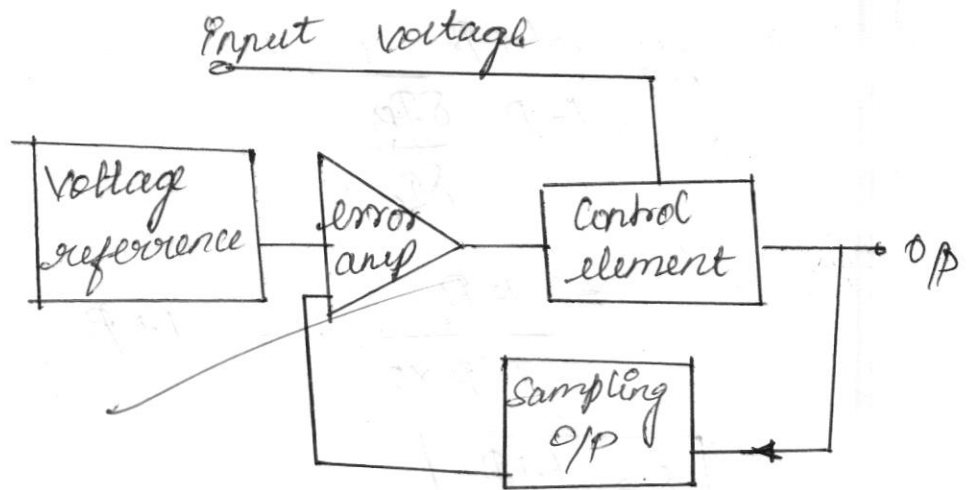
$$\text{the closed loop gain } A_{CL} = 1 + \frac{R_2}{R_3}$$

13) a) Linear mode power supply:

It is a circuit which converts the AC supply in an Equivalent DC voltage, at the required level. The basic building blocks of the Linear power supply,



Basic Building diagram of Linear mode power supply.



1A) BJT & derive the expression for stability

a) factor:

$$s^{\phi} = \frac{\delta I_c}{\delta I_{c0}}$$

$$s^I = \frac{\delta I_c}{\delta V_{BE}}$$

$$s^{II} = \frac{\delta I_c}{\delta \beta}$$

For BJT:

1. Fixed Bias or Base Emitter Bias.
2. collector to Base Bias
3. voltage divider Bias (or) self Bias.

Stability factor: s :-

$$I_B = \frac{V_{CC}}{R_B} I_c$$

$$\frac{\delta I_B}{\delta I_c} = 0$$

$$S = \frac{1 + \beta}{1 - \beta \frac{\delta I_B}{\delta I_C}}$$

$$= \frac{1 + \beta}{1 - \beta \times 0} = 1 + \beta$$

$$S = 1 + \beta$$

$$S'' \Rightarrow S' = \frac{\delta I_C}{\delta V_{BE}} \quad / \quad I_{CO}$$

$$I_C = \beta I_B + (1 + \beta) I_{CO}$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

$$I_C = \beta \left(\frac{V_{CC} - V_{BE}}{R_B} \right) + (1 + \beta) I_{CO}$$

$$I_C = \frac{\beta V_{CC}}{R_B} - \frac{\beta V_{BE}}{R_B} + I_{CO} + \beta I_{CO}$$

$$\frac{\delta I_C}{\delta V_{BE}} = 0 - \frac{\beta}{R_B} \quad \text{to}$$

$$S' = \frac{-\beta}{R_B} \quad / \quad S' = \frac{\delta I_C}{\delta V_{BE}}$$

$$\frac{\delta I_C}{\delta \beta} = I_B + I_{CO}$$

$$\frac{\delta I_c}{\delta \beta} = \frac{I_c}{\beta}$$

$$S' = \frac{I_c}{\beta}$$

$$S = 1 + \beta$$

Advantages:

* circuit is simple.

* Small number of components required

Disadvantages:

* thermal stability is not provided in the circuit.

* stabilization of operating point is very poor.

15) b) Various Biasing BJT switching circuits:

the resistance of R_c for any one of the switching circuit discussed can be calculated by using the specified V_{cc} and I_c level with the explanation.

$$I_c R_c = V_{cc}$$

the transistor h_{FE} (min) value can be used with I_c to determine the minimum β_B level for transistor saturation

Actual transistor current gain can be avoided by using an hFE value of 10.

The resistance R_B for the direct coupled circuit.

$$I_B = \frac{V_S - V_{BE}}{R_B}$$

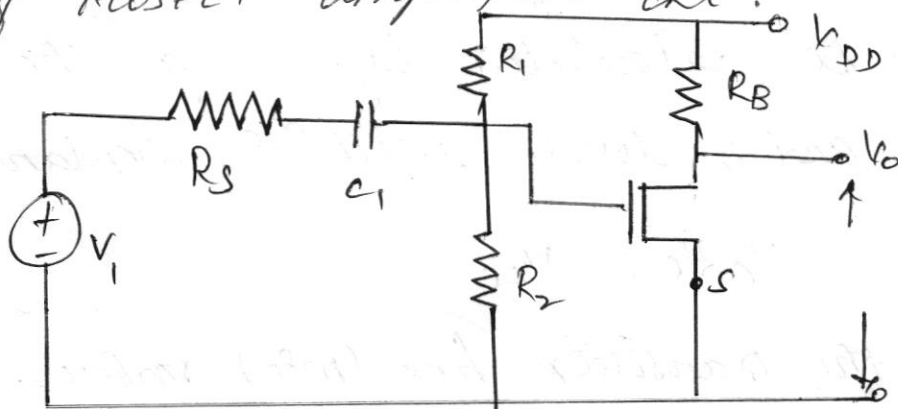
$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

I_C at maximum transistor temperature is unlikely to exceeded $5\mu A$ - So, the maximum resistance.

$$R_B = \frac{0.1V}{5\mu A} = 20K\Omega$$

(B) a) Small signal:

The common source or ground source configuration is the most widely used of MOSFET amplifier ckt.



the source is at ground potential
Hence, the name common source.

the signal from the signal source
is transferred into the Gate of the transistor
through the capacitor.

Input resistance: (R_i):-

the i/p resistance can be determined
from the diagram. the i/p resistance

$$R_i = R_1 \parallel R_2$$

output resistance: (R_o):

the o/p resistance look back into
o/p terminal found by keeping the
i/p voltage to zero

$$V_{gs} = 0$$

$$R_o = R_D \parallel r_{oD}$$

PART A:

1. the Bypass capacitor C_e is connected
in parallel with emitter resistance
 R_e to provide a low resistance path
amplifier AC coupling.

2) Miller's theorem :

The Miller's theorem states that the capacitance impedance, connect b/w i/p & o/p can be C_{eq} .

3) Load Line :

Load line method is used to determine the collector current at the desired collector to emitter.

4) CMRR: Common Rejection Ratio:
the ratio of differential mode gain to common mode gain.

$$CMRR = \left| \frac{A_{dm}}{A_{cm}} \right|$$

5) Emitter resistance :

Emitter resistance is the use of constant current bias.

6) The bias condition mean that is $V_{be} > V_{th}$.
Threshold voltage is called body effect.

- 7) The FET contains of 3 terminal.
Source (S)
Drain (D)
Gate (G) the source of the gate
is called channel.
- 8) It is also called gain is meant that
which a analog amplifier.
- 9) CLM effect is field transistor (FET)
a shorting of the length of the universal
channel.
- 10) JEET is an acronym for Junction effect
transistor
JEET N-channel, P-channel of JEET.



ST. ANNE'S

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Continuous Internal Assessment	III						Unit Test	-				
Register Number	4	2	2	1	1	8	1	0	5	0	2	0
Department	EEE						Semester	VII				
Subject Code	EE 8703		Subject Title		Renewable Energy System							
Date & Session	25.11.2021 / FN						No. of Pages used	27				

A. AZHAGAPPAN	<i>A. Azhagan</i>
Name of the Hall Superintendent	Signature of the Hall Superintendent

Instruction to the Candidate: Put a tick mark (✓) for the questions attended in the tick mark column against each question in Valuation Box											
PART - A			PART - B & C								
Q.No.	✓	Marks	Q.No.	i		ii		iii		Total Marks	
				✓	Marks	✓	Marks	✓	Marks		
					T	D	T	D	T	D	
1	✓	2	11	a	✓	10					12
2	✓	2		b							
3	✓	2	12	a							12
4	✓	2		b	✓	10	✓	10			
5	✓	2	13	a							12
6	✓	2		b	✓	10	✓	10			
7	✓	2	14	a							12
8	✓	2		b	✓	10					
9	✓	2	15	a							12
10	✓	2		b	✓	10					
Total		20	16	a	✓	10					13
			b	✓	10						13
Grand Total	71		Grand Total (in words)								

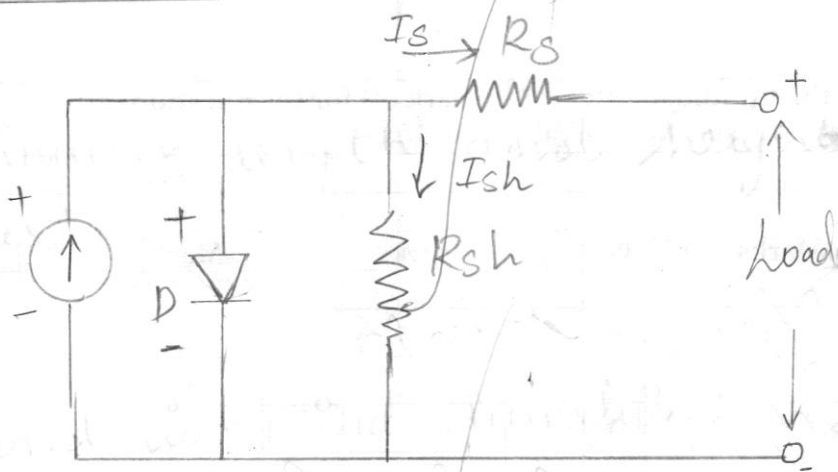
Name of the Examiner	<i>S. Sit</i>	Signature of the Examiner	<i>S. Sit</i>
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PART-A

1. effect of shadowing:

Effect of shadowing is defined as the reactive power Reserving power fluctuation due to the obstruction of the Receiver and Transmitter. The shadowing effect is transmit scattering.

2. equivalent circuit:



3. Different types of PV system:

- * Grid tied
- * Grid / Hybrid
- * Off Grid.

these are different types of PV system

4. Biomass!

Biomass is defined as the plant and animal material to be used as fuel to produce the electric power.

Eg!

Wood, Forest waste waste.

5. Drawbacks of Geothermal energy!

- * Environmental concerns of Green House emission
- * Depletion of the Geothermal plant
- * Land required for Geothermal thermal
- * Investment cost is high

6. Surge Tank!

Surge Tank is used for storage of water. It is an Hydro power plant. It limits the pressure of flow water.

7. Water Turbine!

There are two types of water turbine are

Impulse turbine

- Pelton turbine
- Turgo turbine

Reaction turbine!

- Francis turbine
- Kaplan turbine *

9. Maximum length and height of ocean waves!

The first wind is wind power. Second is wind duration. The factor affecting fact the wind power is to the maximum waves, without changes wind duration. *

8. Tidal energy!

Tidal energy is the same Hydro power it the tidal produce the electric power. The gravitation of the sun and moon also on earth.

There two wind + tide energy

- * Tide range
- * Tide stream

10. factors affecting fuel cell!

- * Pure
- * pressures
- * Temperature
- * Load current.

these are the affecting the fuel cell.

PART - B

b.a)

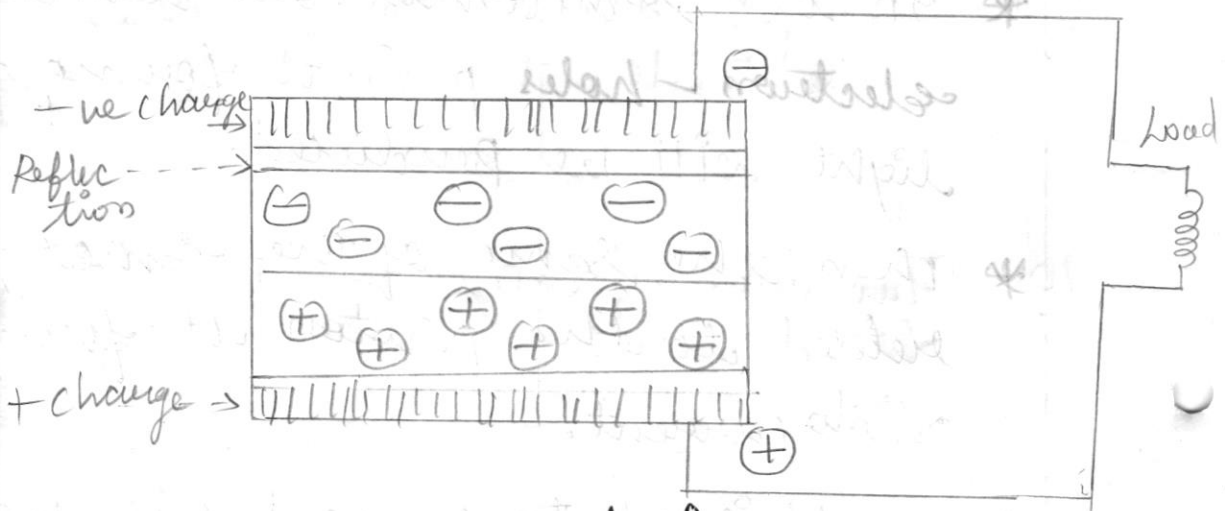
Solar photo voltaic System.

- * Solar energy is converted to the electrical energy.
- * This process is the solar photo voltaic.
- * Solar photo voltaic is the sun light emit the photons.
- * It is the process is the normal emitting of energy.
- * Photo voltaic effect is the semiconductor is emit by the energy in sunlight.
- * to the electricity will produced.
- * It is an the renewable energy. It energy directly to sun light.
- * The energy is transmitted to the system.
- * In the photo voltaic is the semiconductor emitted by diode.
- * It is an the through photons it will produces the internal electric field.
- * The force to the electric field It will current flow through the tunnel.

Photovoltaic effect:

- * In the semiconductors devices if an electron-hole pair is formed the light will be produced.
 - * Then the same of the energy is released to the photon it forms the photocurrent.
 - * In this photocurrent will be passed through the forward dark current.
 - * There is no voltage at the output voltage.
 - * Only on the photocurrent is the system that is called photovoltaic effect.
 - * Then the photocurrent value is to be I_{pv} subtracted to the photocurrent and dark current.*
- $$I_{pv} = I_D - I_p$$
- * It is an effect is called photovoltaic.
 - * The electron emits the current in the system.

Working!



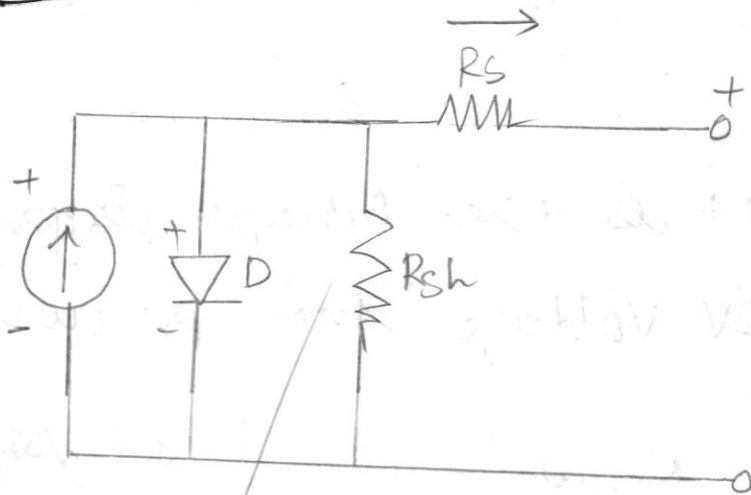
Solar photo voltaic system.

- * The solar photo p system is the consist of the Semiconductor devices.
- * It is the emit the sun light the electric electric energy is produced.
- * The light is emitter to the current is produced. then the output is the current.
- * Then the light emitter "dopping" of the energy.
 - o Negative charge is lightly dopping
 - o positive charge is heavily dopping
- * The photocurrent to the energy is the produce to the output current.
- * There is not output Voltage and Diode voltage.

$$I_{PV} = I_P - I_0 (e^{V/kT} - 1) I_P$$

e^V is the Boltzmann constant, I_P is photon current, k is the temperature of I .

- * The equivalent circuit of the solar cell is the current flow of the energy.
- * There is no voltage and diode.
- * Semiconductor is connected in parallel of source.
- * Resistor is connected to the load.
- * The resistance is decrease, circuit is short-circuit.



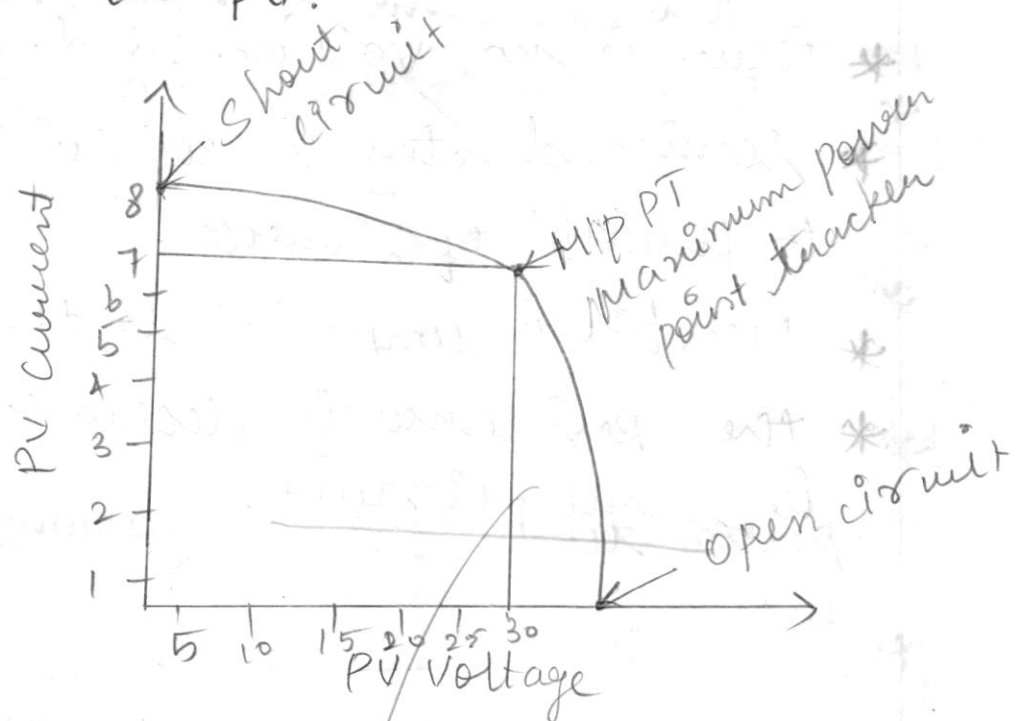
Equivalent circuit of PV

- * The current is flows and there is no voltage.
- * Then the resistance is increase the circuit Open circuit.

* The voltage is output. The resistance increase.

* there is no current. the flow of photon current either the electric field.

* there are the equivalent circuit of the PV.



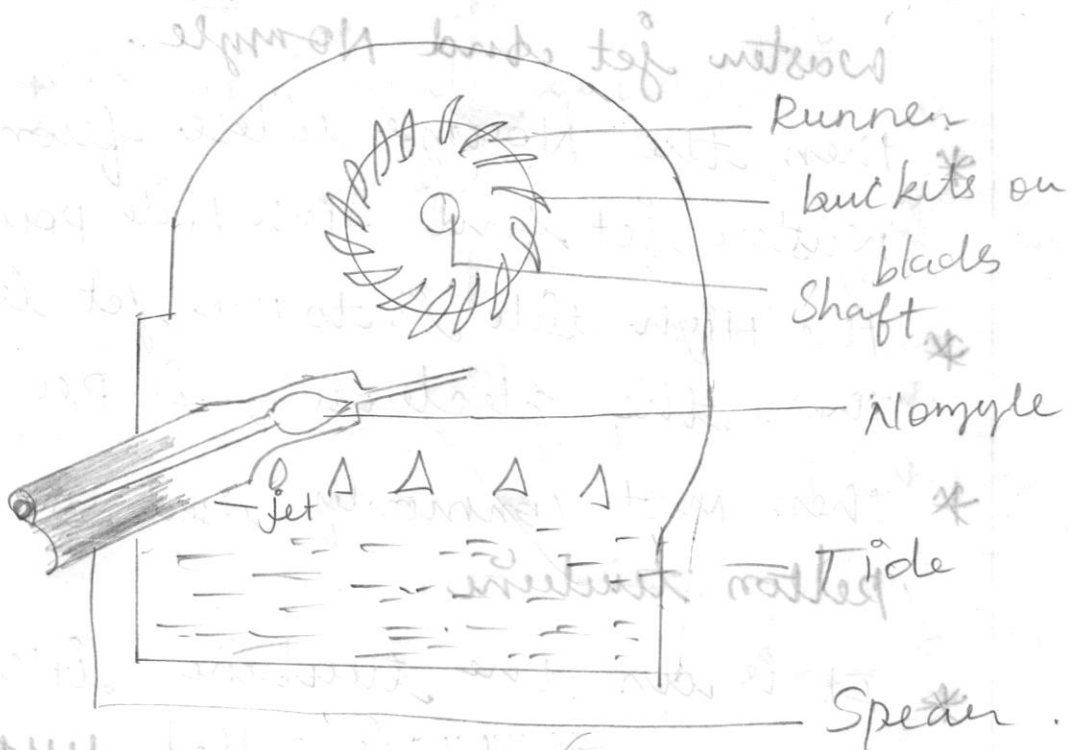
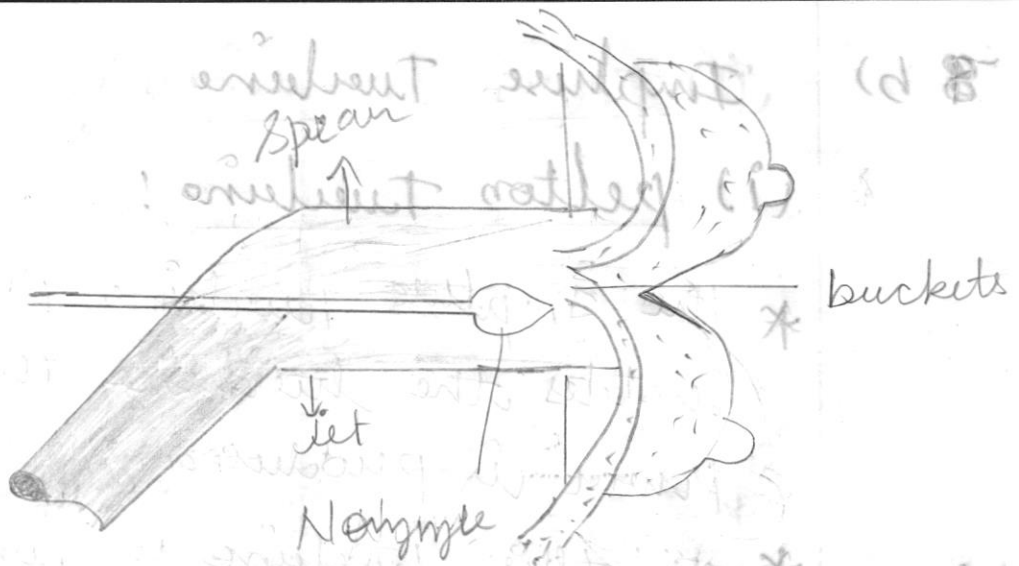
* It is the graph is plotted between PV voltage and PV current.

8 b)

Impulse Turbine

(i) pelton turbine!

- * the impulse turbine working is the rotates the turbine the electric power is produced.
- * In this turbine is consists of the water jet and nozzle.
- * then the nozzle release from the water jet and high tide power.
- * the high tide is to the jet is rotate then the electricity is produced.
- * then most commonly used to the pelton turbine.
- * It is in the turbine it's rotates runner that is called runner.
- * It is the runner is connected to the turbine shaft.
- * And the ~~nozzle~~ ^{nozzle} is the release from the buckets and blades.
- * The four buckets rotates its series combination of the circular shape.
- * It is made up of cast iron, bronze or stainless steel.

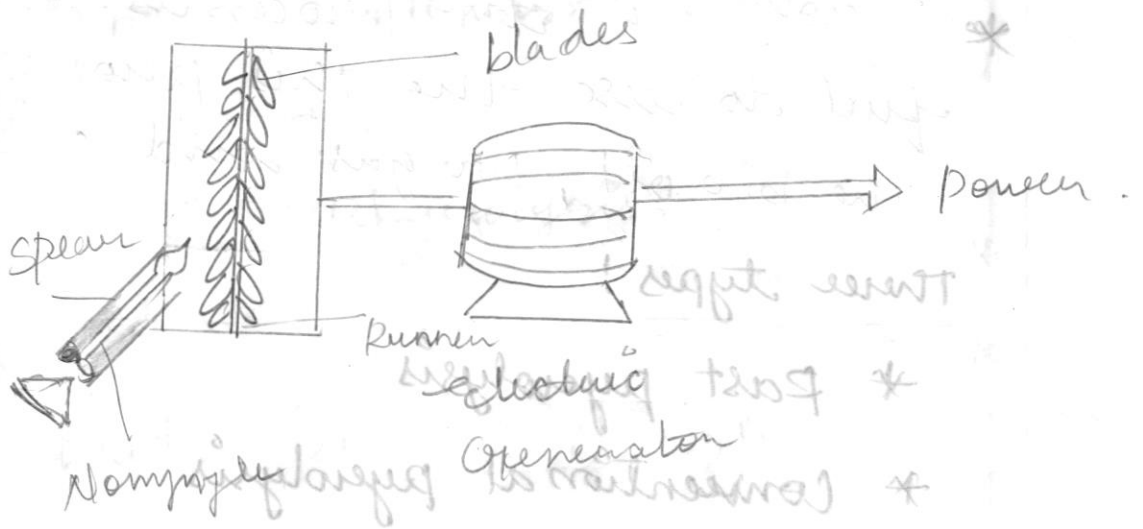


* This is known as hemispherical blades.

* It is the runner is connected to the turbine and generator electric is produced.

* It is the Pelton turbine of the water ticks plant.

(ii) Turgo Turbine:



* Turgo turbine is similar to the Pelton turbine.

It is blades is under the rotating and electricity produce.

* The Turgo turbine compare to the higher power and speed of the Pelton turbine.

* It is power consumption is high.

* That is the Turgo turbine

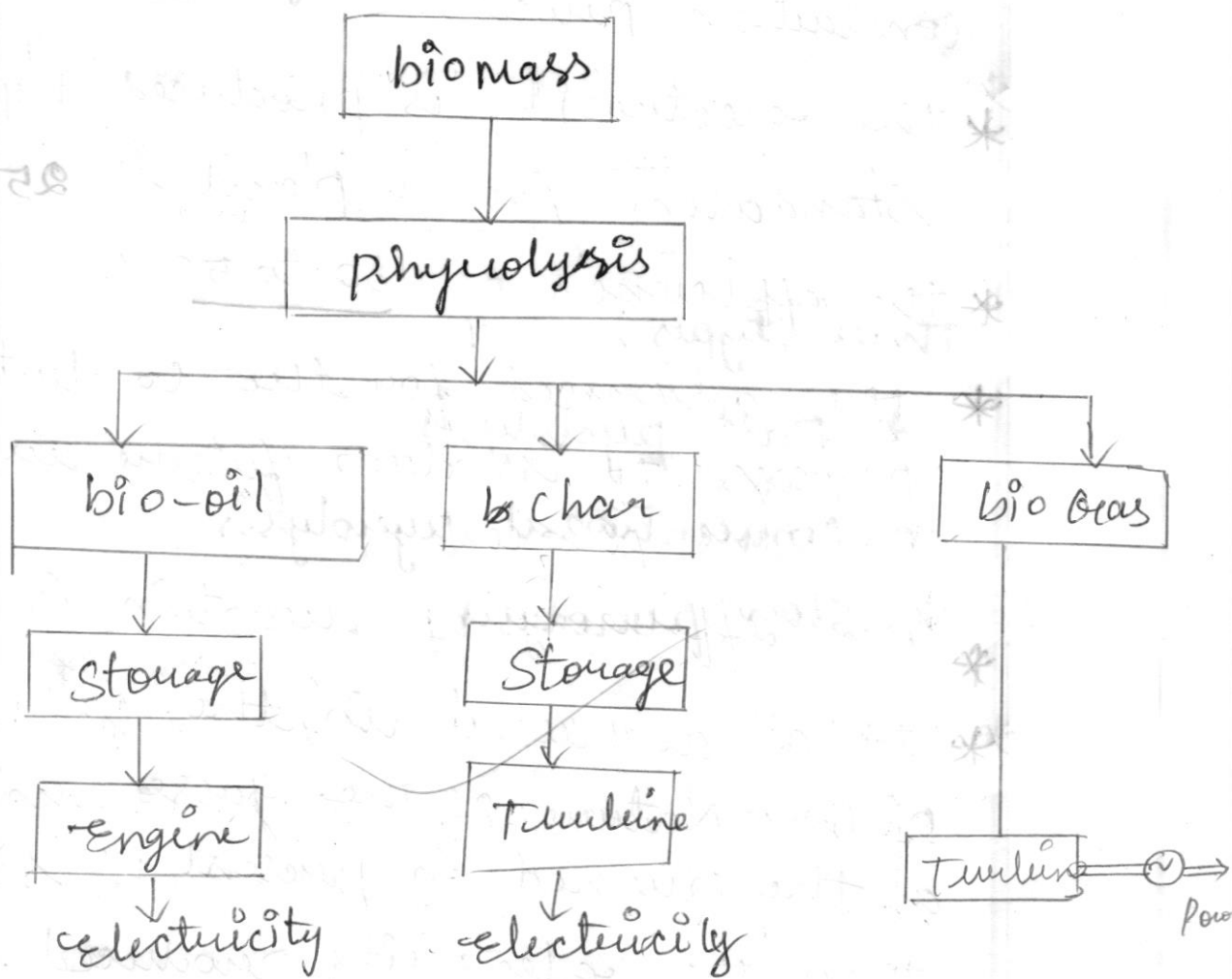
7 b)

(i) Pyrolysis;

* Pyrolysis is the process is Natural fuel to use the this process are the bio oil, bio gas and char.

Three types :

- * Fast pyrolysis
 - * Conventional pyrolysis
 - * Slow pyrolysis.
- * Fast pyrolysis is high heat and high temperature transfer heat. 60% of bio oil, 20% char, 20% of bio gas.
- * Conventional pyrolysis is the process through gases of the air the low heat transfer.
- * Slow pyrolysis is the slow than fast 10 time of slow of fast pyrolysis.
- * Flash pyrolysis is the similar to the fast pyrolysis.
- * It is an high temperature heat is transferred.
- * That is pyrolysis
- * The efficiency is 80%.



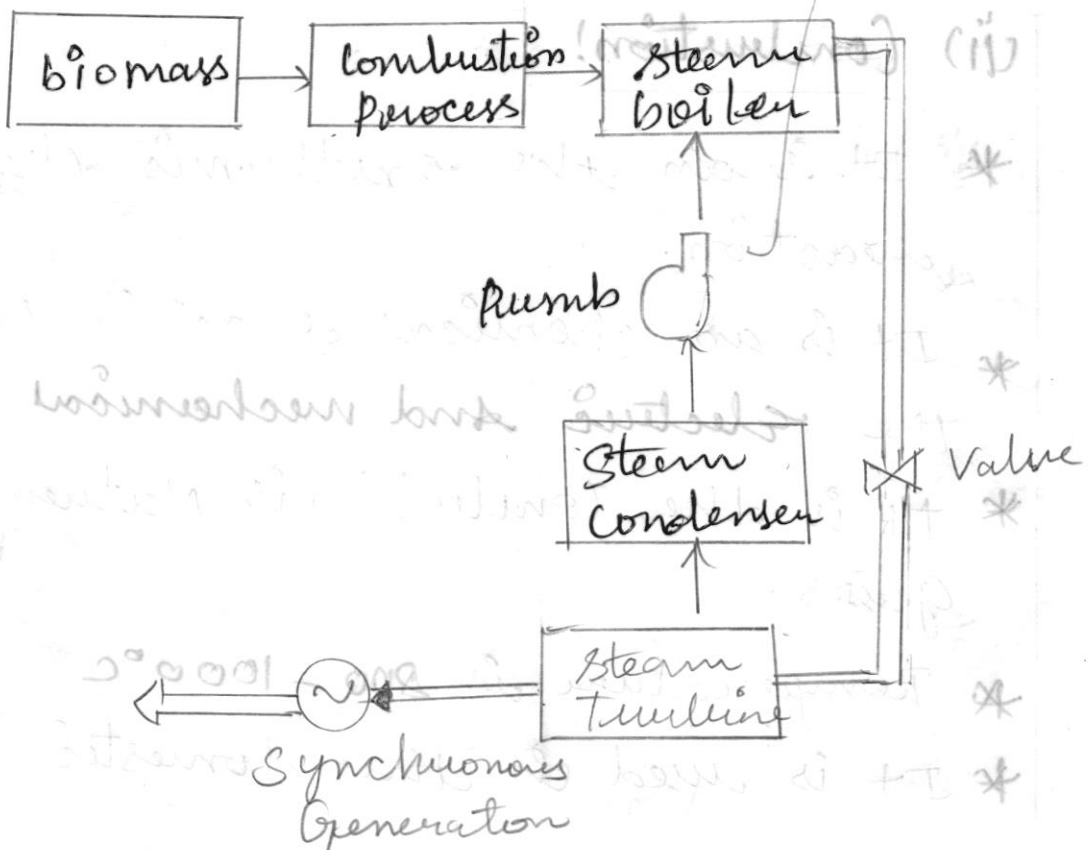
process of pyrolysis.

(ii) Combustion!

- * It is an the exothermic chemical reaction.
- * It is an chemical reaction produce the electric and mechanical energies.
- * It is the combustion is Natural of Gasses.
- * Temperature is $800 - 1000^{\circ}\text{C}$
- * It is used in the domestic and

commercially used the gases of the combustion process.

- * The electricity is produced by the standalone power plant is 25 to 50 MW.
- * The efficiency of 30 to 50%.
- * The advanced for the combustion process. In modern future are used in this process.
- * The efficiency of reaction is 50 to 70%.
- * It all are used in the bed system of the nature of the gases and fuel as the burned in presence of air. Then the electricity produced.



9. b) OTEC!

OTEC - ocean Thermal Energy Conversion.

* Ocean thermal energy conversion is also the ocean thermal energy it is the process of converting the electric energy.

* OTEC consists of the deeper part of the cold and shallow one is the sea of the energy.

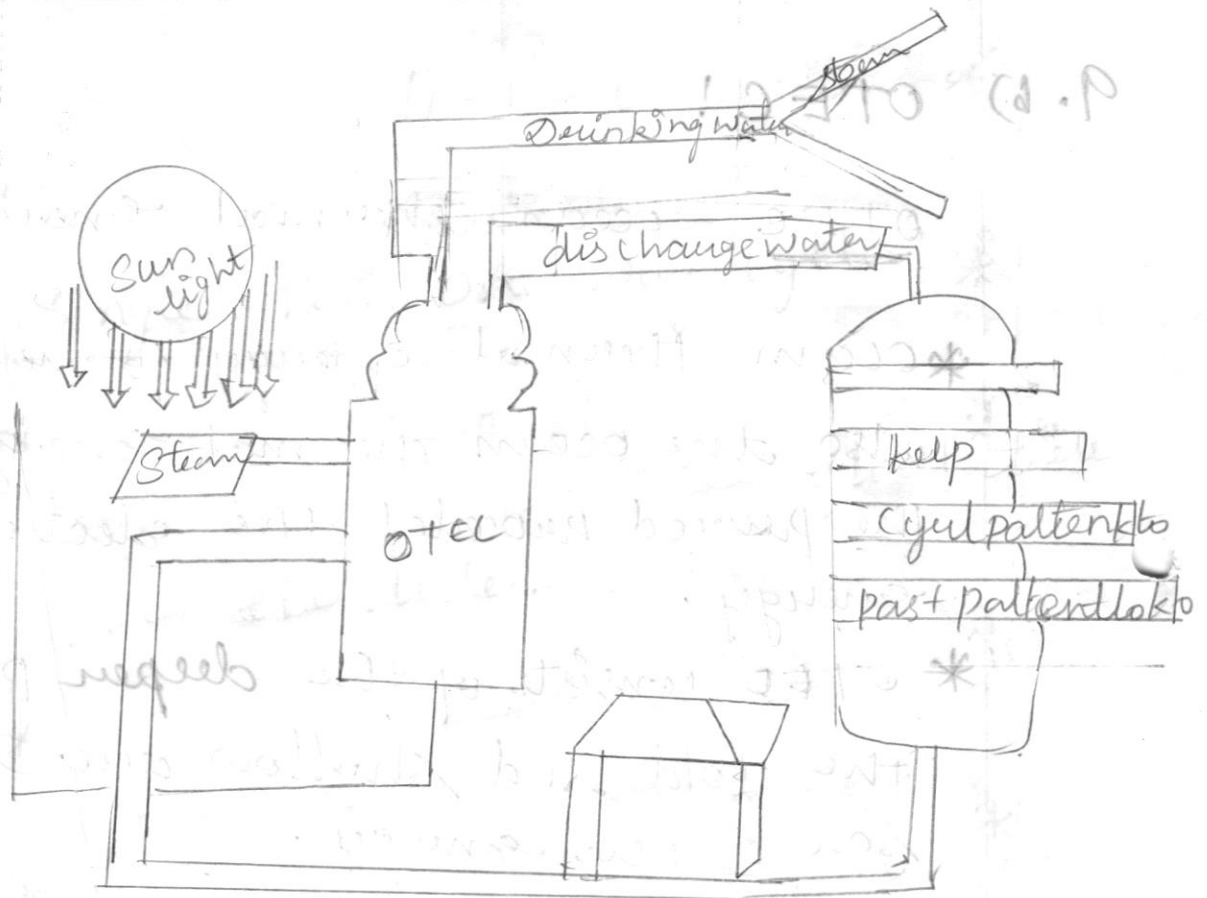
* The cold sun the engine energy move to the one to another of the sea level.

* Then the deeper is to the low temperature the water cannot deeper to sea.

* It is the ocean thermal energy is the conversion of the tide energy of the sea.

* It is a heat is to the sunlight does not enter to the sea.

* It is use of deeper that is the level is indicate.



OTEC

- * It has temperature difference.
- * The temperature difference to the deeper it also different of the efficiency.
- * Then the temperature is increase also efficiency is increase.
- * It is the process of the Ocean Thermal to power is generation.
- * The deeper to the shallow part of the sea is the bed of the system.

* It is the one most renewable energy.

* It is the ~~to~~ reduce the world demand power

* It is reduce the world power demand. one it is installed.

* The efficiency of the ocean thermal energy is the 100%.

* but is Initial cost high.

Advantages!

* It is pollution free

* Renewable.

* High efficient

Disadvantages!

* Initial cost is high.

PART - C

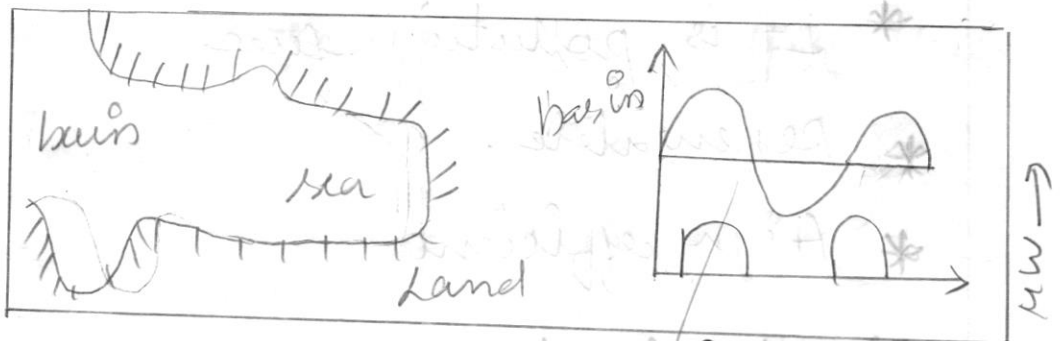
11. b)

Single-basin one way cycle! *

* Single basin one way cycle is the energy from the flood tide and ebb tide.

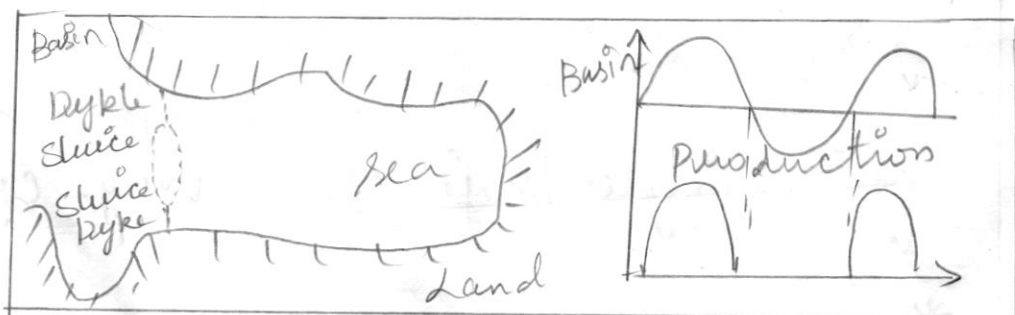
* The basin is to the either the flood tide and ebb tide.

* The energy will be produced.



Single basin basin

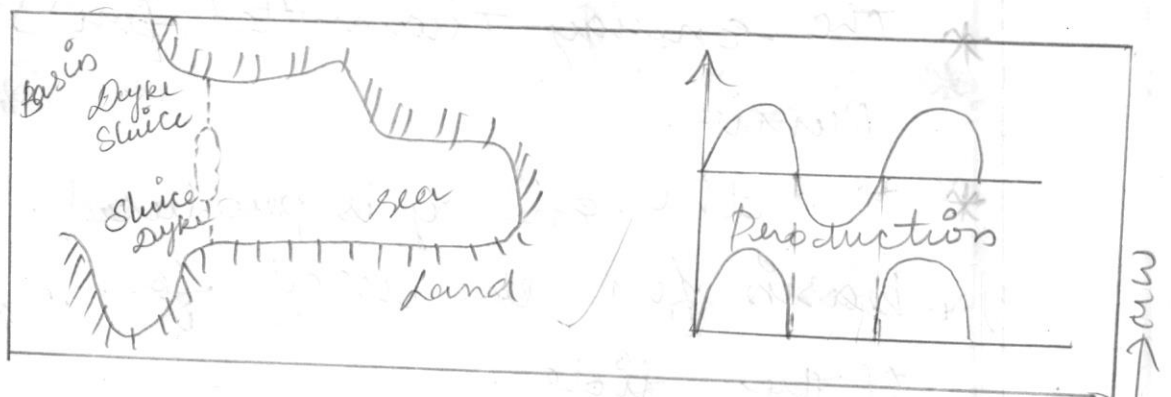
* The basin is the constructed to the land use of flood and ebb.



Single basin one way cycle.

Single basin Two way cycle!

- * Single basin two way is the basin is used to the flood tide and drains the ebb tide.
- * The energy through the two ways one is the flood tide.
- * The drains used the ebb tide.
- * Basin is constructed for the one basin to another basin of flood tide.
- * Its action is the water raise and fall.
- * The high energy of the water is used for the flood tide.



Single basin Two way-cycle

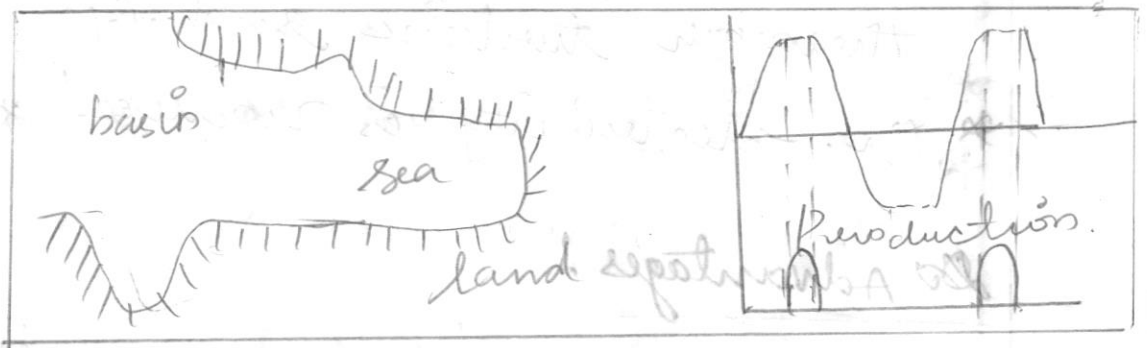
- * The used for one basin controlled by two tide.

Single basin ^{pump} Storage system:

- * Single basin the energy through the basin used.
- * The basin carried from the energy flood tide and ebb tide.
- * High energy of the water in sea and to move.
- * The pump is used for water move one basin to the another basin.
- * The water in the pump.
- * And also the release to the rising flood ebb.
- * It is the power to tidal then to the turbine and power generator.
- * The energy transmitted for the pump.
- * Then the energy is produced. And basin for the used for control of the tide.
- * To sea water is used for tidal energy.

Double basin system!

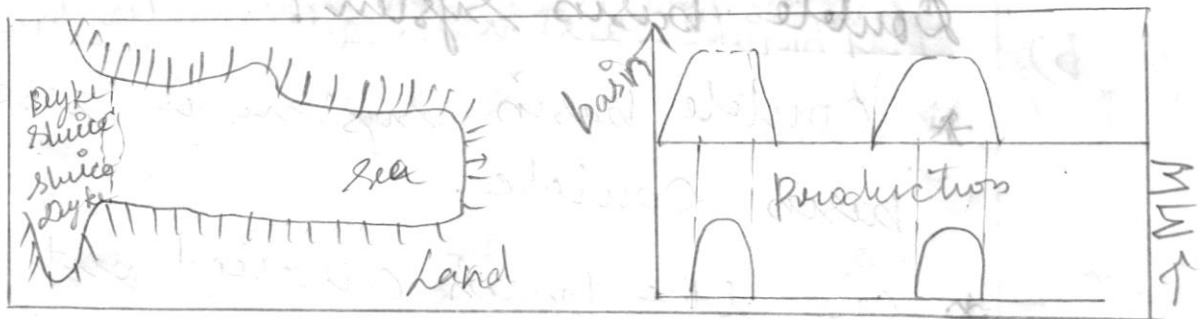
- * Double basin system is used the basin double.
- * One the basin carried out the flood tide. Another one is the ebb tides.
- * It is similar to the single basin system.



Double basin system.

Double basin one way cycle!

- * It is the two basins either flood tide drains the ebb tide.
- * That constructed to the double basin one way cycle.
- * two basins controlled by the one way.



Double Basin one way cycle.

Double basin pump storage:

- * It is the tidal energy stored to the pump.
- * The energy to the pump water through turbine and generator.
- * The electricity is produced.

Advantages:

- * It is pollution free.
- * It is renewable.

Disadvantages:

- * It is cost is high.
- * It is installed in land center.

10. b) Hybrid system?

- * Hybrid system is define as the
It is the continuous process of the
Production electricity.
- * Hybrid means two power system
is coupled.
- * there is an continuous production
electricity.
- * that is called Hybrid system.

Need for Hybrid system?

- * Hybrid system world wide useful
of the system.
- * It is need for the sun light only
on the daytime there night time
is there no production of electricity
- * the the solar and wind to Hybrid
The continuous energy do.
- * there no power demand in day to
day life.
- * so need for the Hybrid system.
It is production is Increase.

- * The Hybrid system used for the many country countries.
- * It is useful of the consumption of power.
- * They need for the Hybrid system.
- * They reduce the power demand.
- * It is used useful one of the Hybrid system.
- * In the future they use of the Hybrid system.

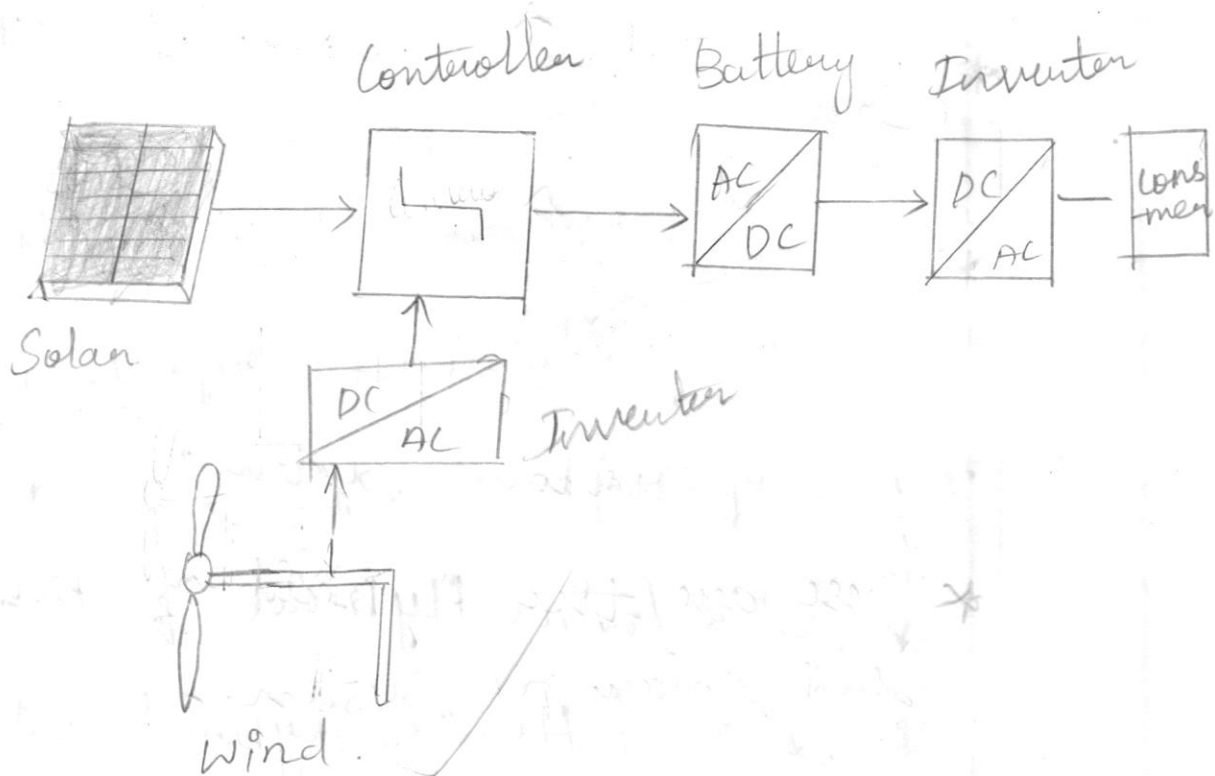
Types of Hybrid system:

- * Solar / wind Hybrid system
- * Solar / Hydro Hybrid system
- * Solar / tidal power system.
- * PV system / Geothermal Hybrid system

Solar / wind Hybrid system:

- * It is an solar is only a day time power consumed.
- * Wind is the air consumed for the all time.

- * The setting of the solar power is the day time start up with Night time wind start up
- * It will the continuous process of the solar energy is produced.
- * then the reduced. The power demand.

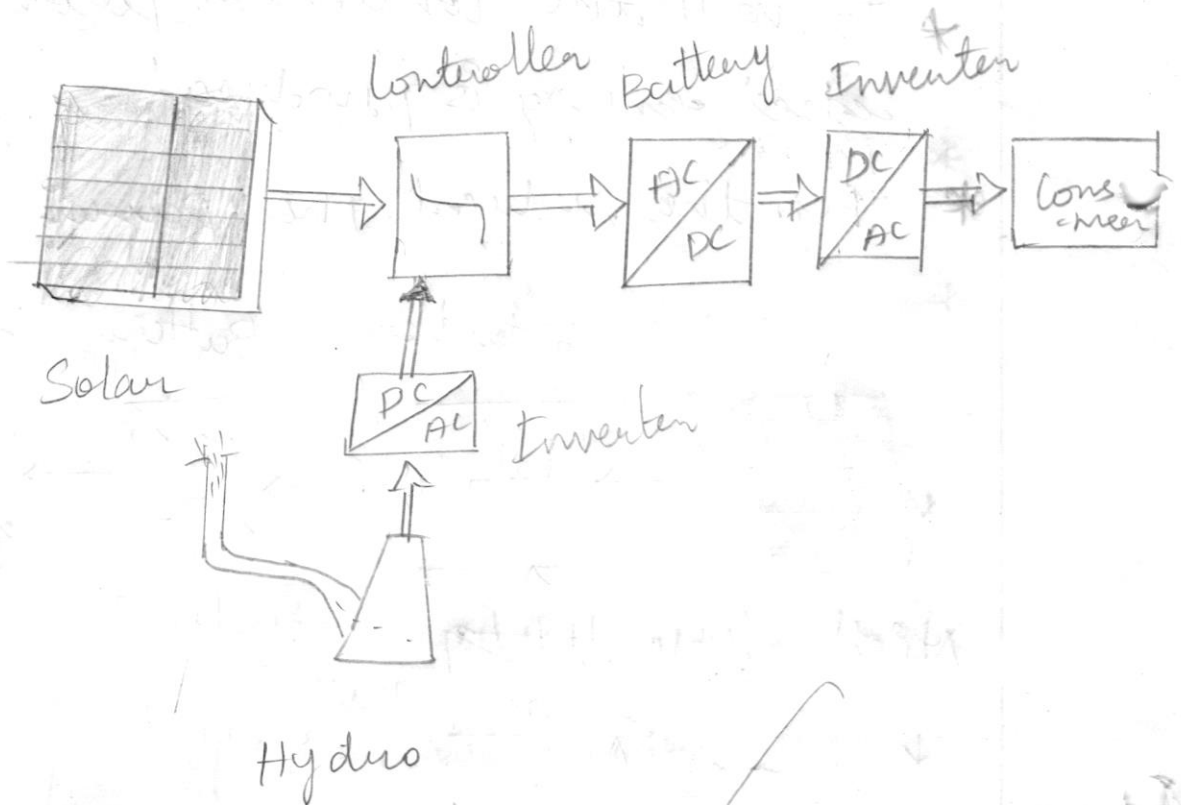


Solar / Hydro Hybrid System!

- * Solar and Hydro Hybrid the system.
- * Gravitational force to the Hydro It is the used to the coupled of the solar.

* Its construction is same as the w/s Hybrid system.

* PV Solar same as working.



* These are the Hybrid of the Hydro and solar PV system.

Solar / Hydro Hybrid system!